Nonlinear description of transversal motion in a laminar boundary layer with streaks

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June 27, 2012

Streaky flow in flat plate boundary layer

STREAKS

Streamwise long, spanwise thin structures, HS Down-LS Up.

Streaky flow in flat plate boundary layer

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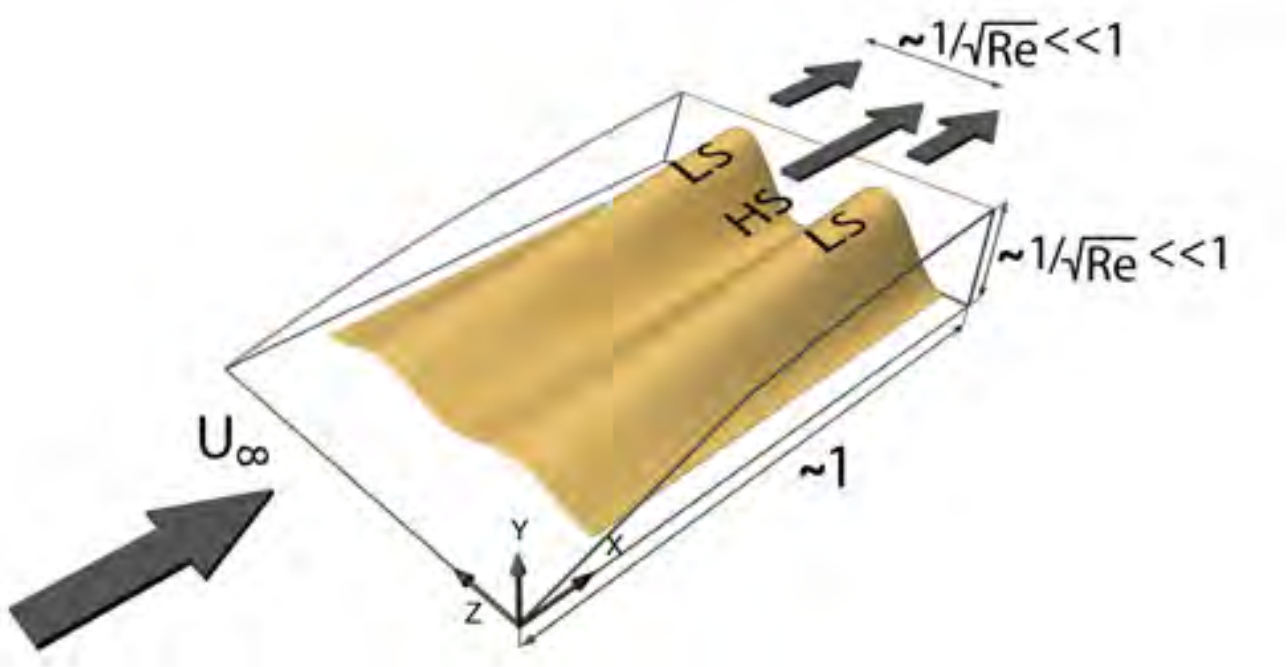

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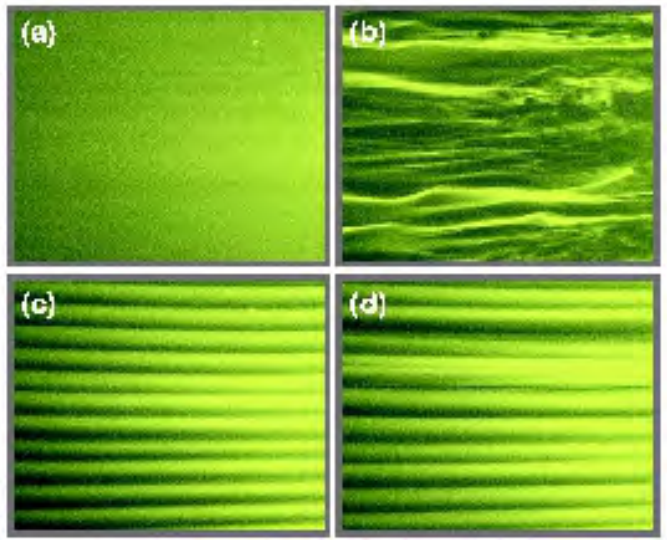
3D Streaky BL more stable to TS perturbations than 2D Blasius BL
Transition to Turbulence delayed, theoretically and experimentally

without TS **with TS**

STREAKS off

STREAKS on



Fransson & Talamelli & Brandt & Cossu PRL 2006

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Edge states in boundary layer (low speed streaks)

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Motivate computation of high amplitude streaks

Computing streaky boundary layer flow

Laminar nonlinear high intensity streaks

Computing streaky boundary layer flow

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Tipically compute using DNS and nonlinear PSE

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- DNS

  Very CPU costly

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- **PSE**
  Consistency problems

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$\Rightarrow$ **3D RNS** Reduced Navier Stokes equations - **Fletcher 1990**

Computing streaky boundary layer flow

Laminar nonlinear high intensity streaks
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$\Rightarrow$ **3D RNS** Reduced Navier Stokes equations - **Fletcher 1990**

Simplified boundary layer like formulation - **BRE's**
Streaks induced by Free Stream Perturbations.

Wundrow & Goldstein 1998

Leib & Wundrow & Goldstein 1999,2001

Ricco & Luo & Wu 2011

Computing streaky boundary layer flow

Laminar nonlinear high intensity streaks
Tipically compute using DNS and nonlinear PSE

- **DNS**
  Very CPU costly

- **PSE**
  Consistency problems

$\Rightarrow$ **3D RNS** Reduced Navier Stokes equations - **Fletcher 1990**

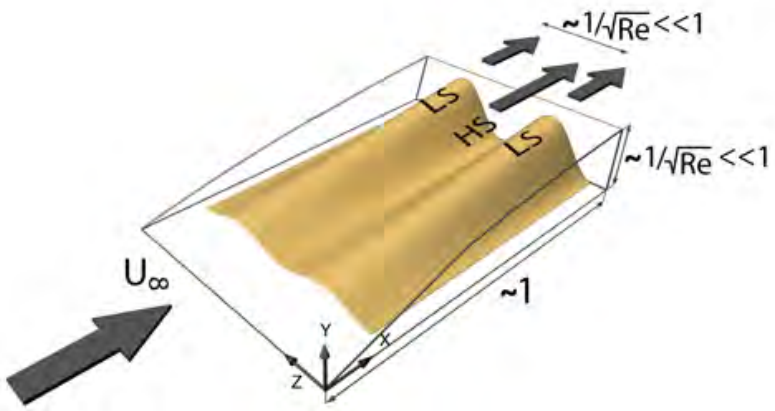
Simplified boundary layer like formulation - **BRE's**
Robust and fast computation
RNS compute the downstream evolution and flow pattern of high amplitude streaks

RNS 3D

$Re = U_\infty L/\nu \gg 1$

x scaled with L
y, z scaled with $\delta \sim L/\sqrt{Re}$
u scaled with $U\infty$
v, w scaled with $U_\infty/\sqrt{Re}$
p scaled with ρU_∞^2

2 short scales y, z + 1 long x



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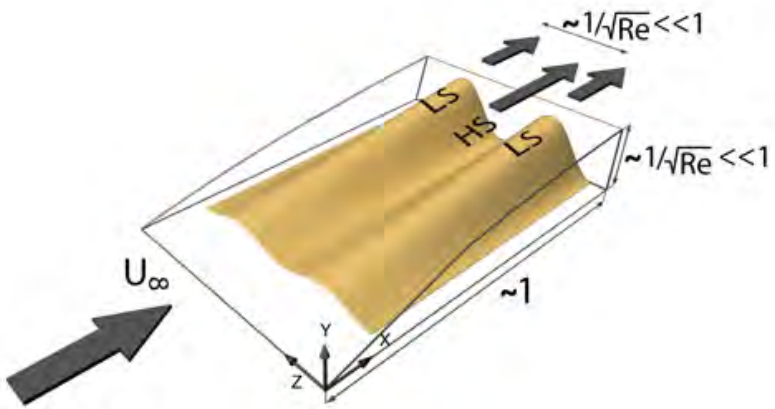

2 short scales y, z + 1 long x

$$(u, v, w) = (u, v, w) + \frac{1}{Re}(u, v, w)_1 + \cdots$$

$$p = p + \frac{1}{Re} p_1 + \cdots$$

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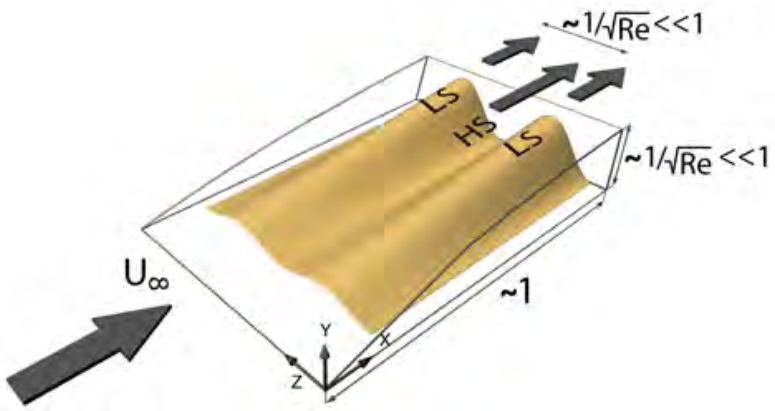

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$$(u, v, w) = (u, v, w) + \frac{1}{Re}(u, v, w)_1 + \cdots$$

$$p = p + \frac{1}{Re} p_1 + \cdots$$

Standard 3D BL: 1 short scales y + 2 long x,z !!

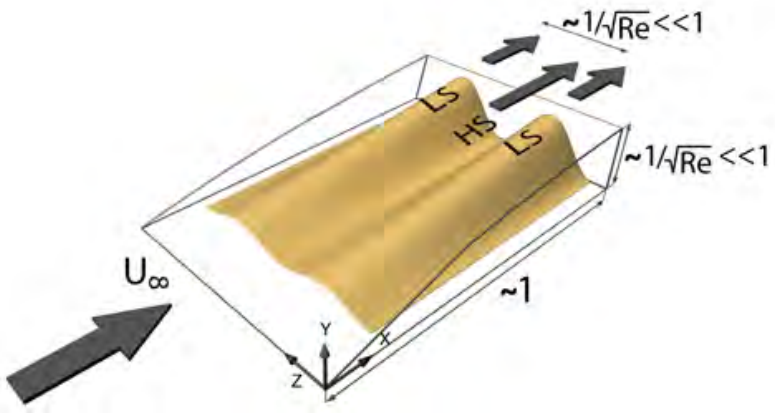
Schlichting BLT 1979

RNS 3D

At first order, M-y, M-z:

$$\left.\begin{array}{l} p_y = 0 \\ p_z = 0 \end{array}\right\} \Longrightarrow p = p_0(x)$$

Inviscid flow



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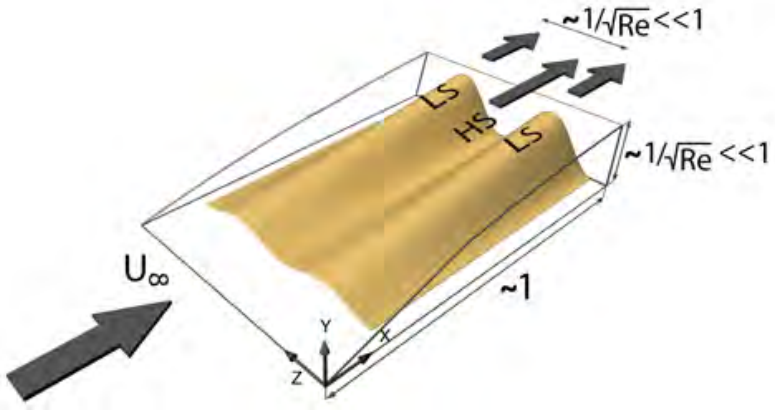

Inviscid flow

And then, M-x, Cont, M-y, M-z

RNS 3D eqs.

$$u_x + v_y + w_z = 0$$
$$uu_x + vu_y + wu_z = -p_{0_x} + u_{yy} + u_{zz}$$
$$uv_x + vv_y + wv_z = -p_{1_y} + v_{yy} + v_{zz}$$
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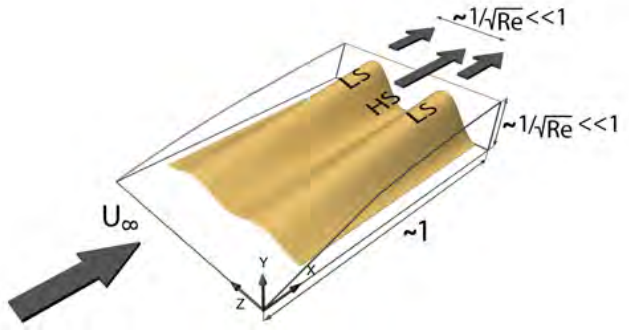

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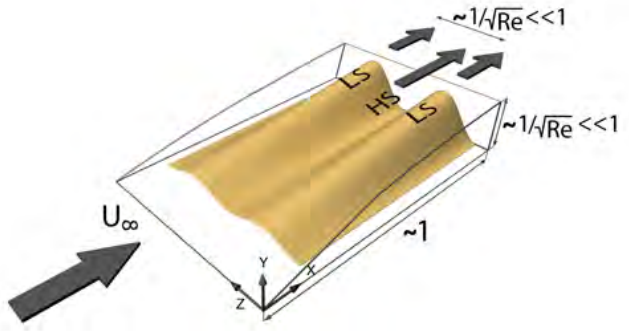

- p_1 **coupled** with u, v, w, 2nd order y-z momentum required.

RNS 3D

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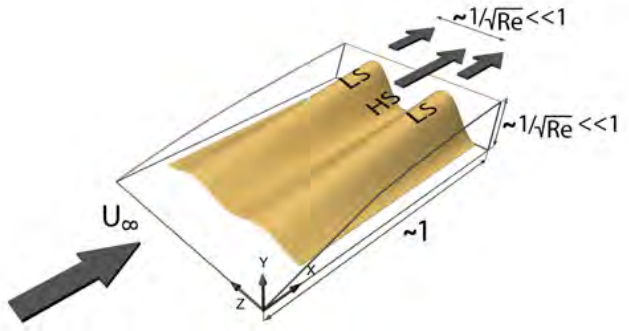

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Boundary conditions

RNS 3D

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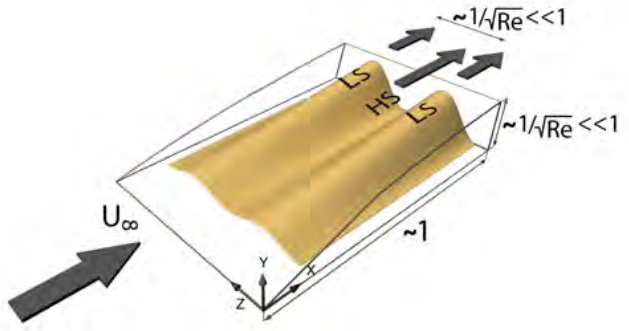

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- $u = v = w = 0$, at $y = 0$.

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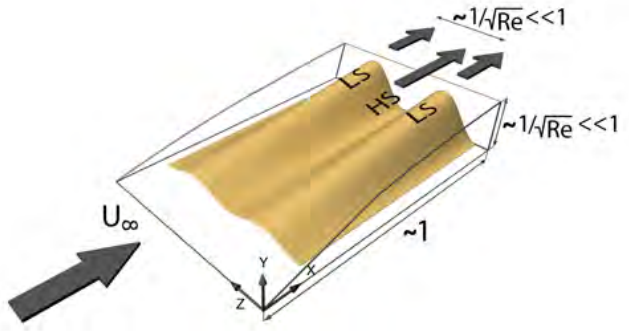

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- $u = v = w = 0$, at $y = 0$.
- $u = u_\infty$, $w = 0$, $v - < v >_z = 0$, as $y \to \infty$.

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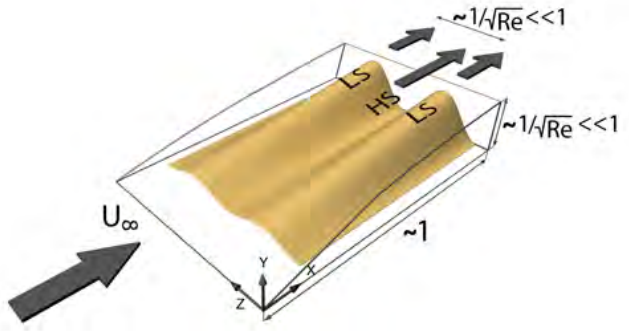

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- $u = v = w = 0$, at $y = 0$.
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- Periodicty in z.

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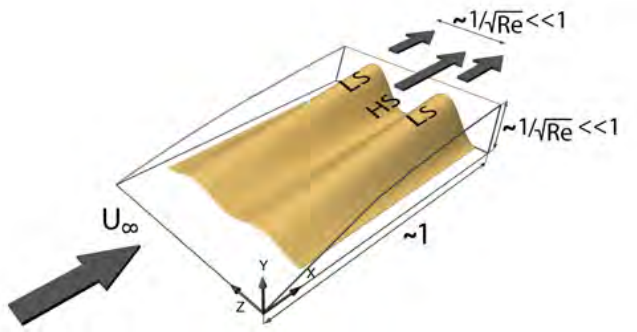

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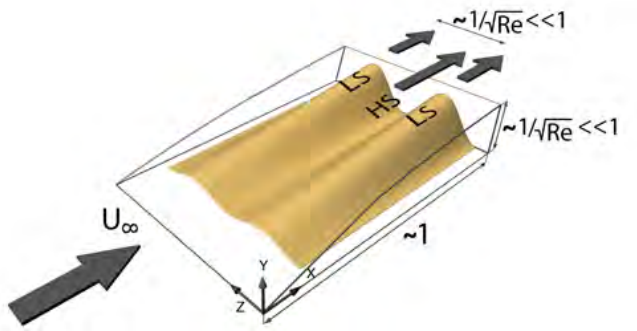

- $Re \to \infty$ Asymptotic States.

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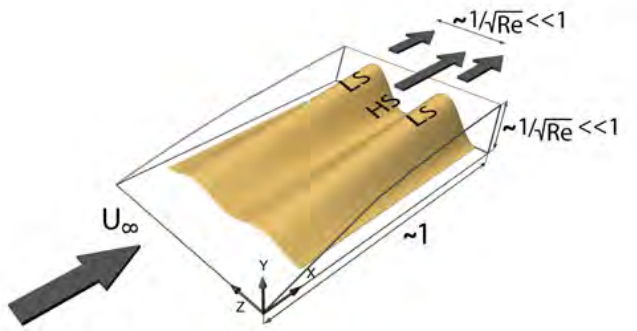

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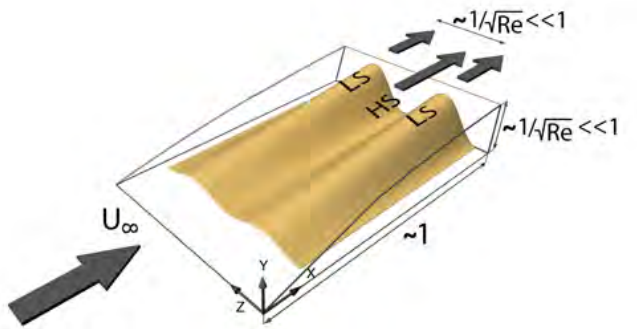

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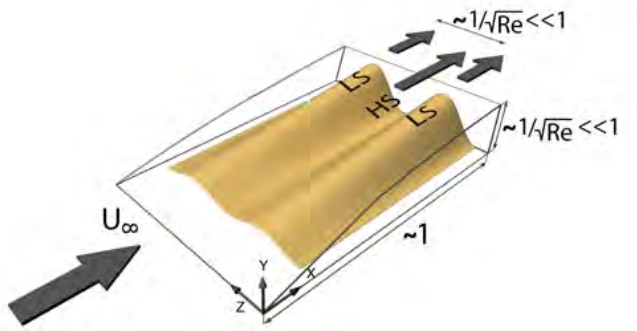

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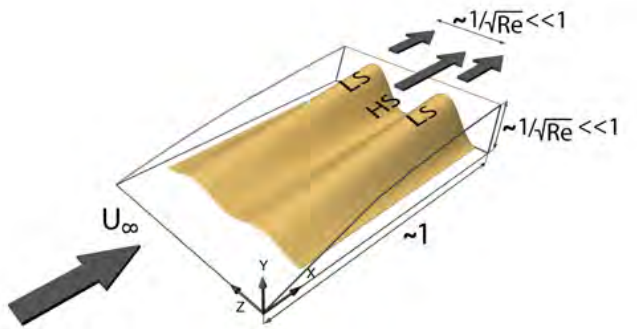

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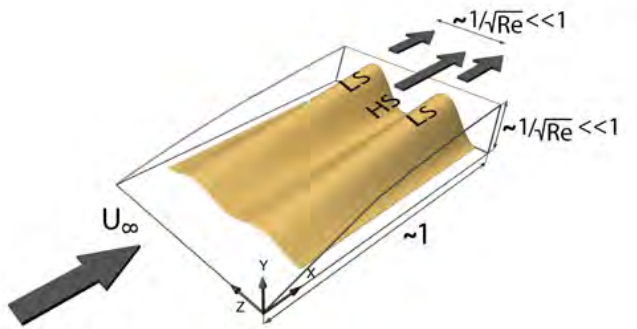

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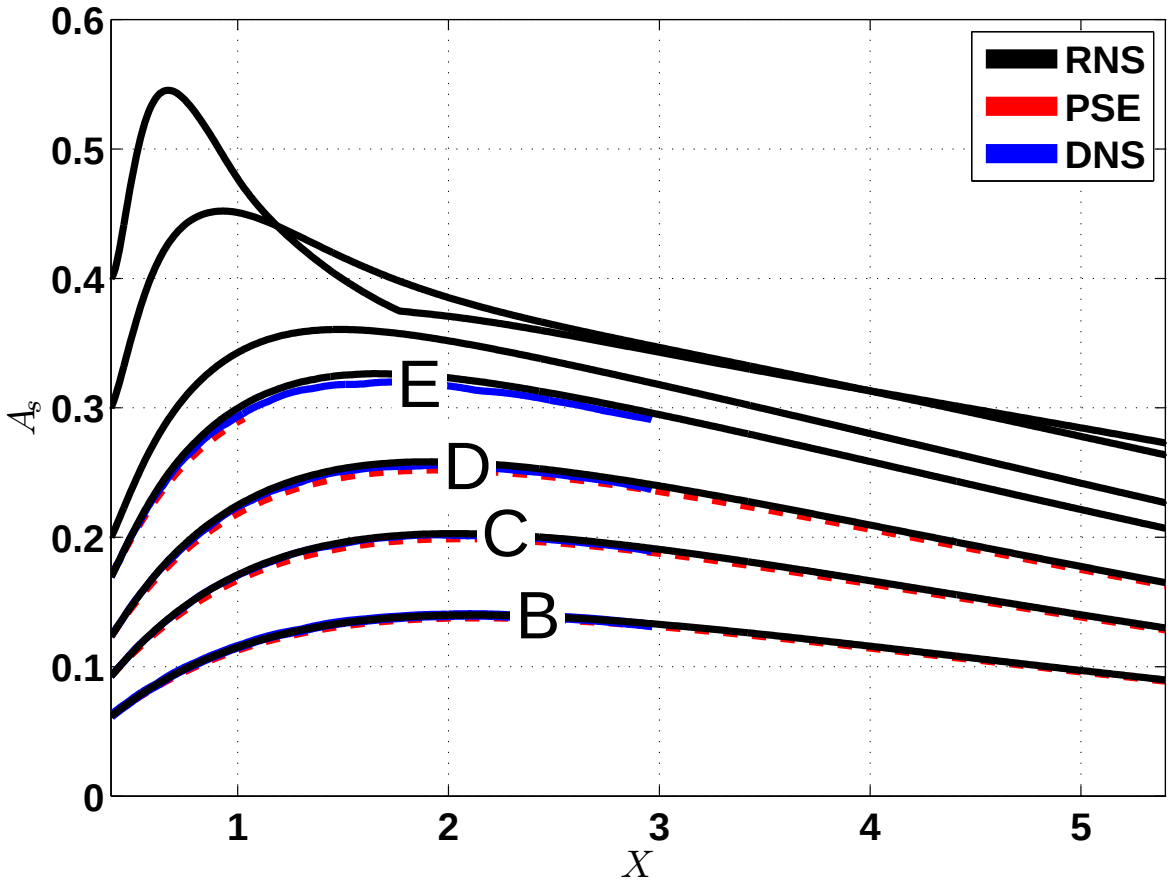
- 2nd order BDF x marching, 2nd order finite diff. in y and z.
- Improved bc at $y \to \infty$ (**Higuera & Vega JFM 2009**).
- Faster than DNS, more robust than NPSE.

RNS 3D results: Nonlinear Streaks



DNS: Cossu & Brandt PoF 2002, EJMB/F 2004

NPSE: Bagheri & Hanifi PoF 2007

RNS 3D results: Motion in transversal plane

Downstream evolution of the particles trajectories.

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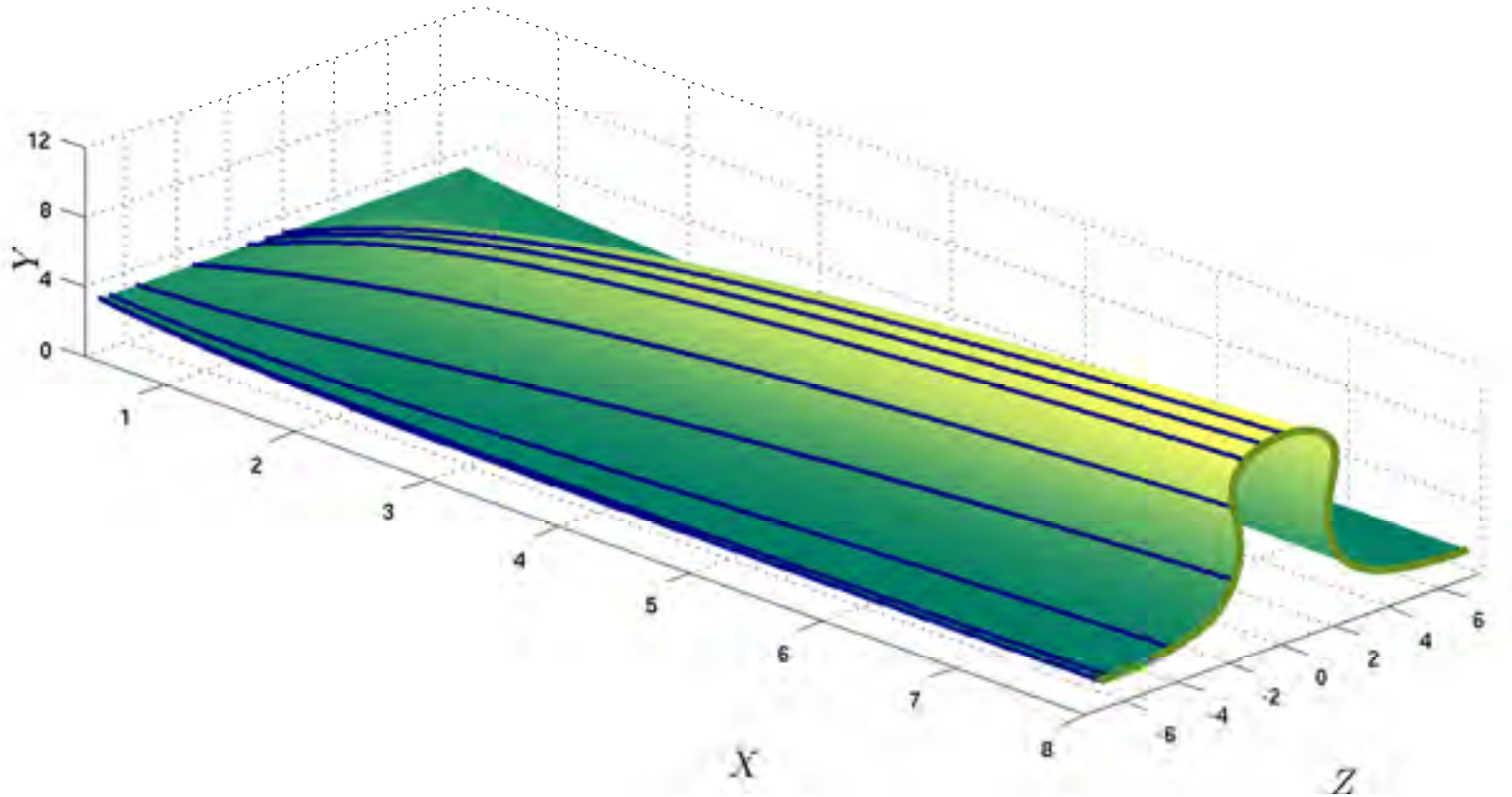
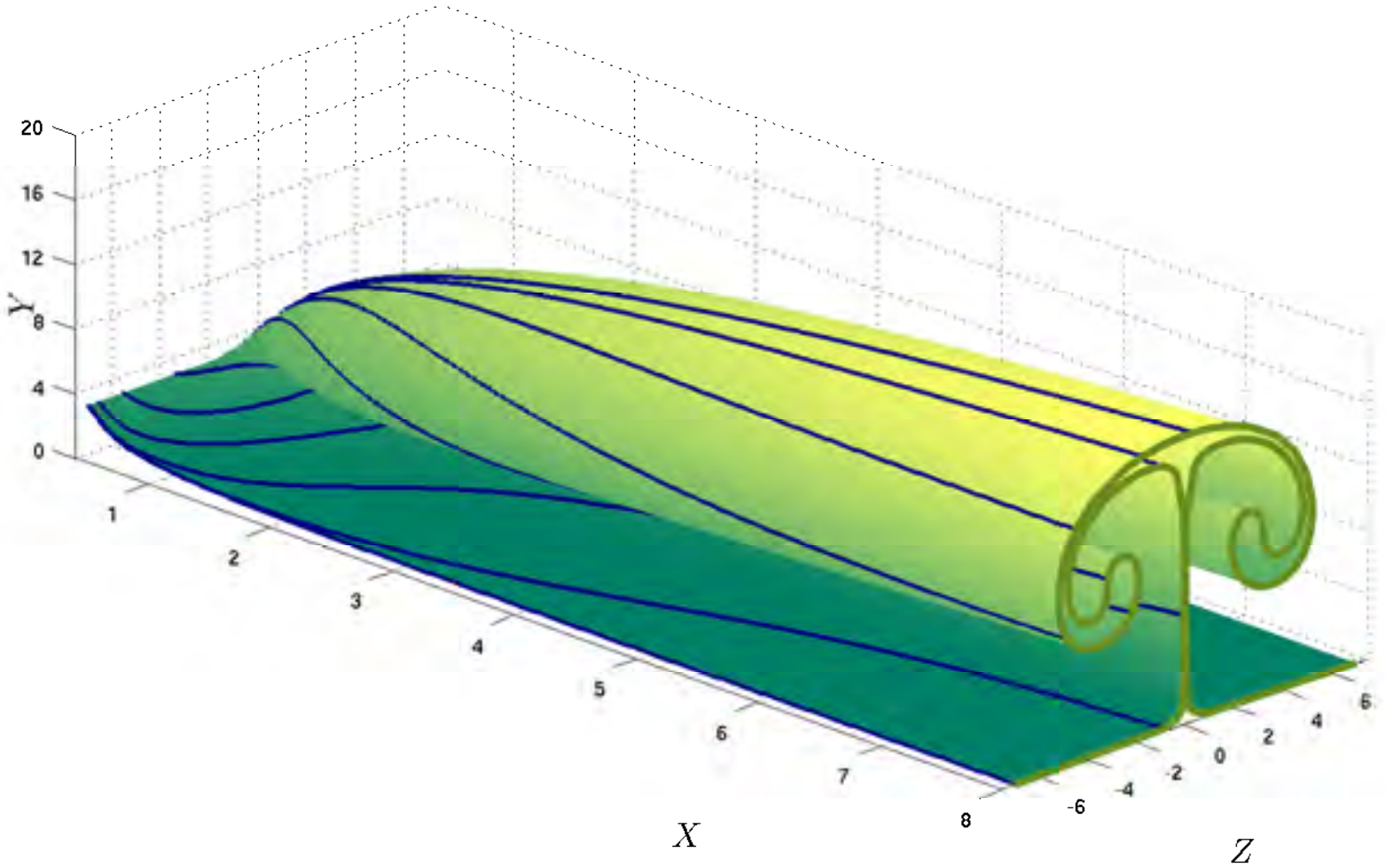


$\mathbf{A_{s0} = 0.1}$**, departing from line** $\mathbf{x = 0.4}$**,** $\mathbf{y = 3}$

RNS 3D results: Motion in transversal plane

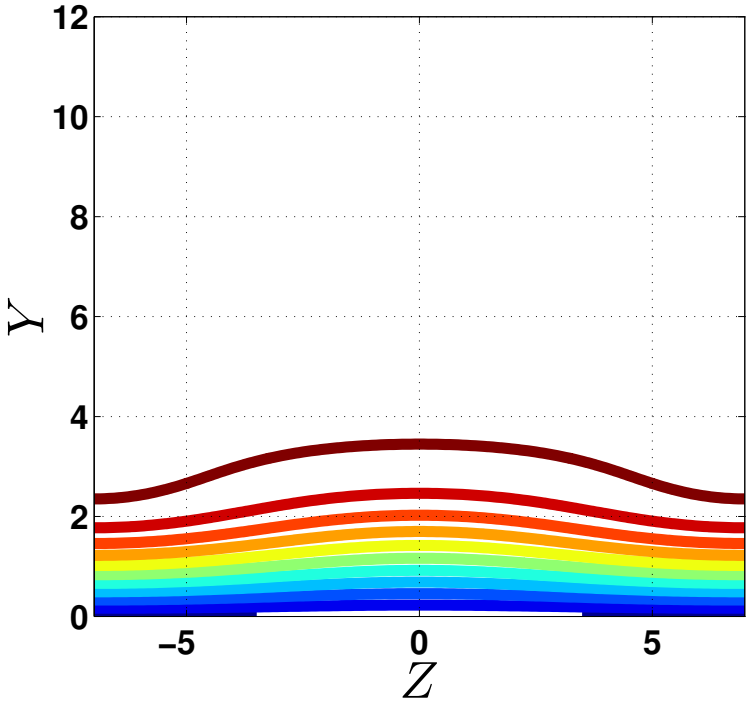
Downstream evolution of the particles trajectories.



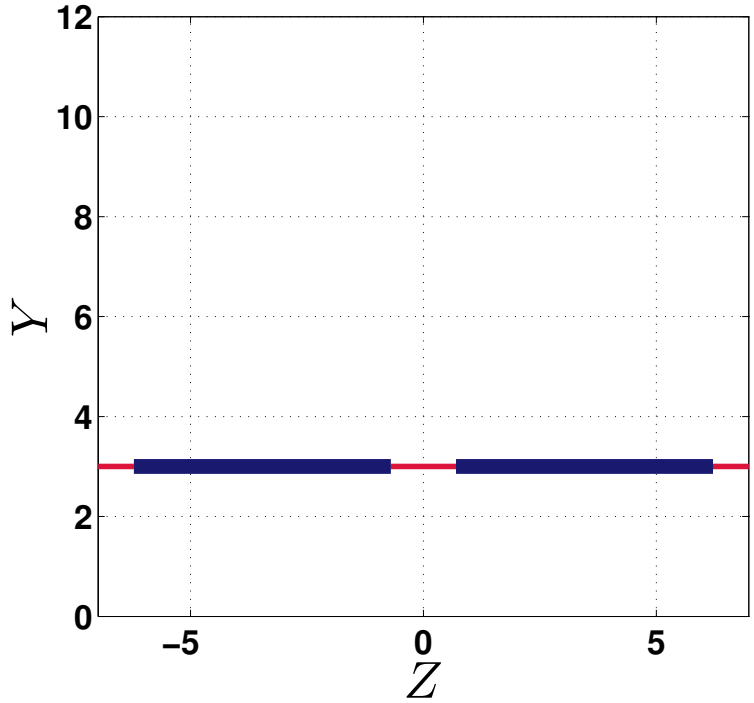
$A_{s0} = 0.4$, departing from line $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Characteristics of the streaks $A_{s0} = 0.1$, $x = 0.4$



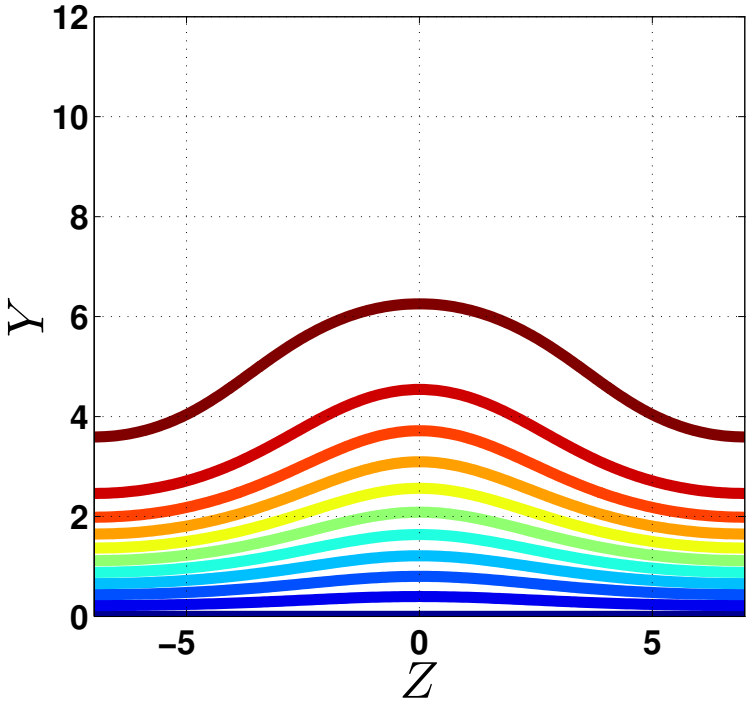
u constant streamwise contour lines



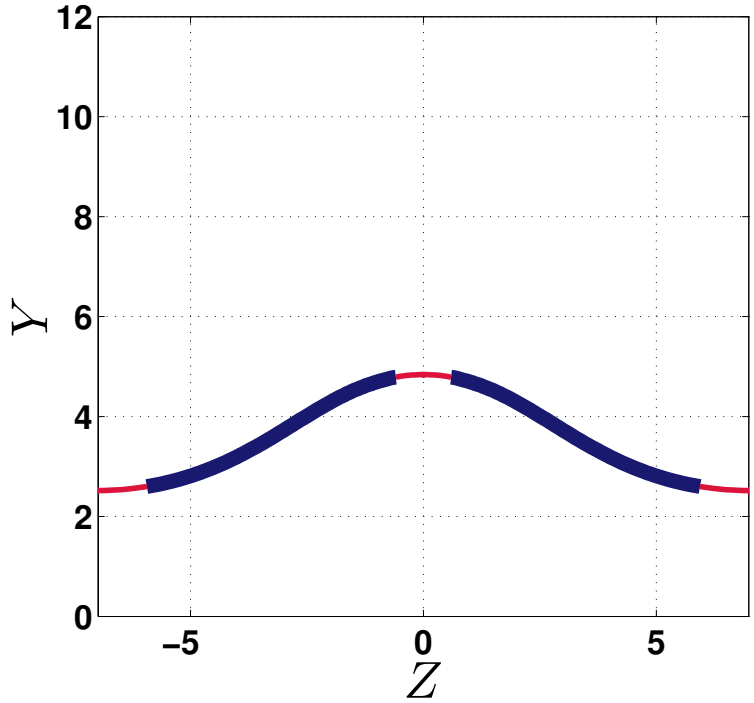
sections of particles trajectories starting $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Characteristics of the streaks $\mathbf{A_{s0} = 0.1}$, $\mathbf{x = 1}$



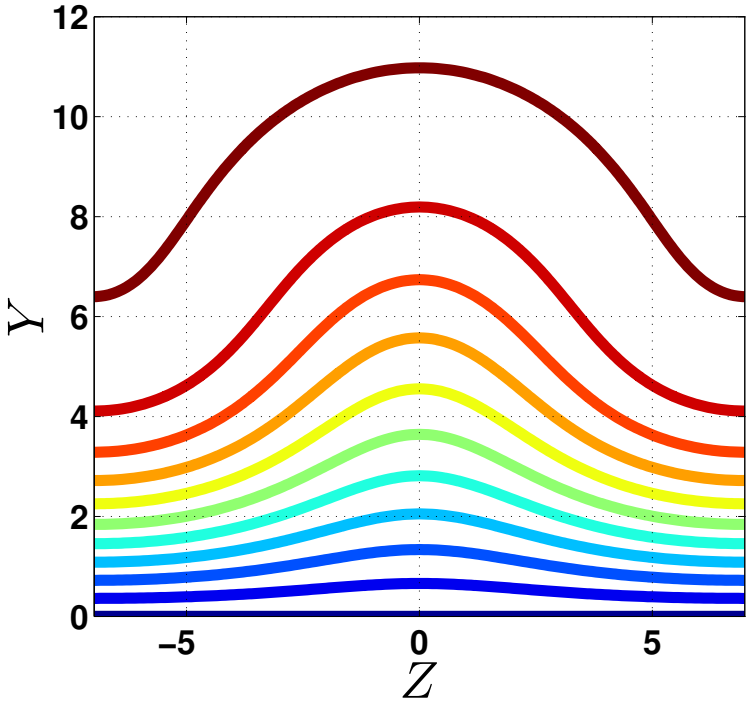
u constant streamwise contour lines



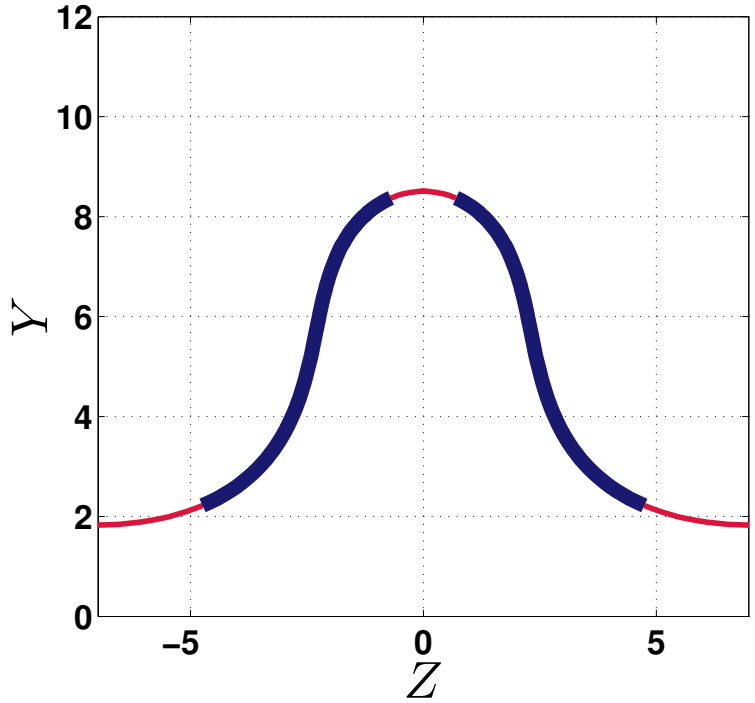
sections of particles trajectories starting $\mathbf{x = 0.4}$, $\mathbf{y = 3}$

RNS 3D results: Transversal structure

Characteristics of the streaks $A_{s0} = 0.1$, $x = 3$



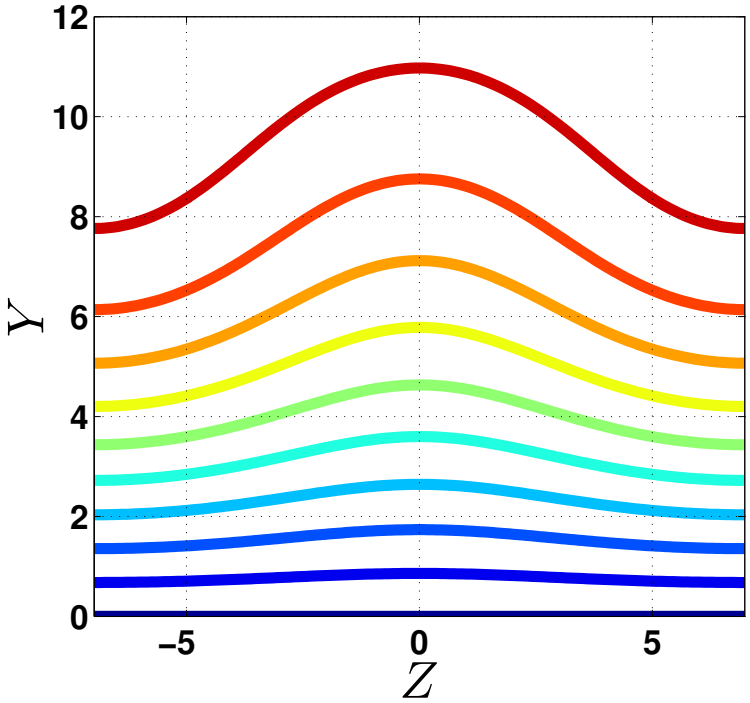
u constant streamwise contour lines



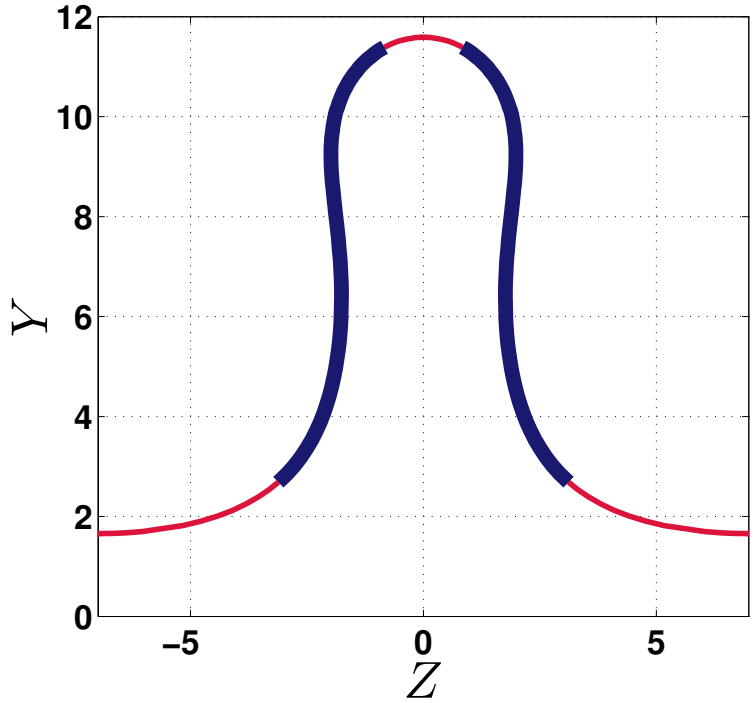
sections of particles trajectories starting $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Characteristics of the streaks $A_{s0} = 0.1$, $x = 7$



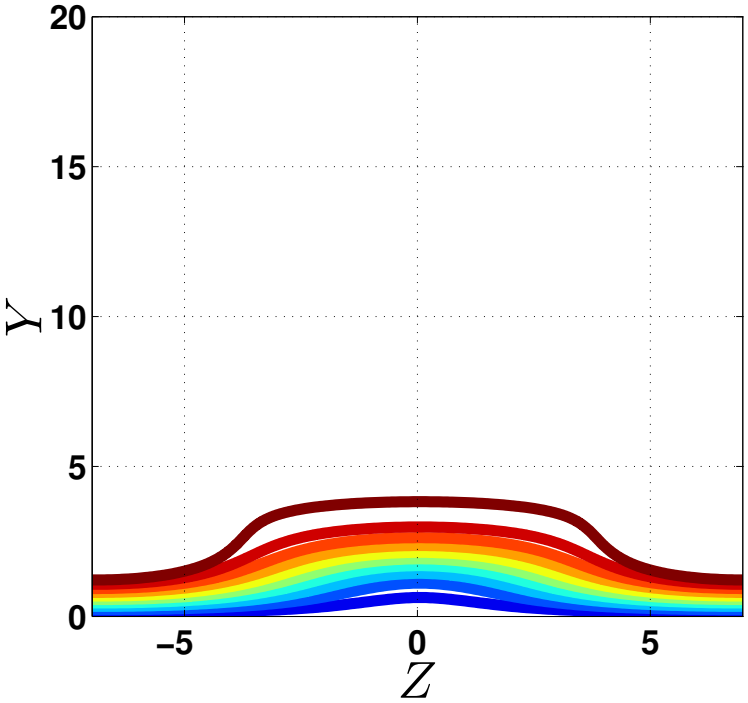
u constant streamwise contour lines



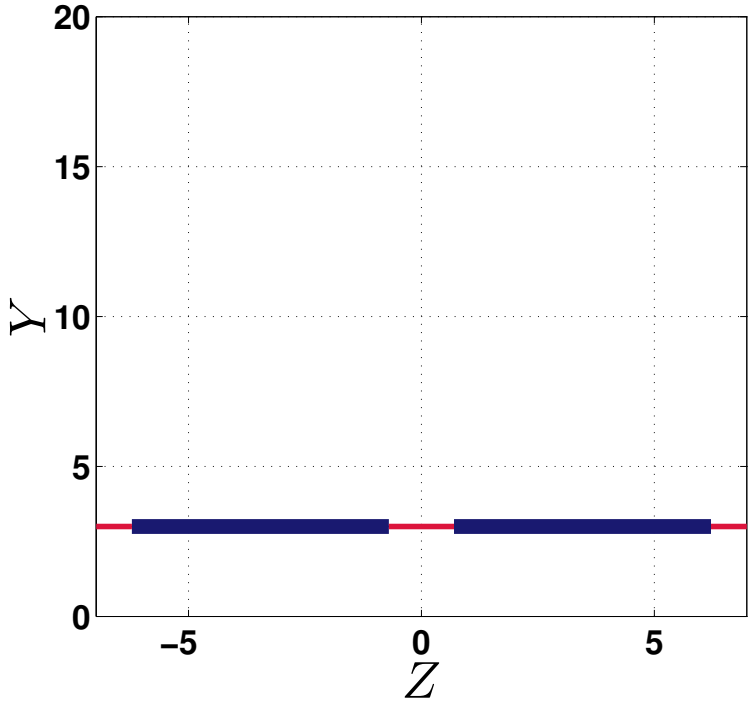
sections of particles trajectories starting $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Characteristics of the streaks $A_{s0} = 0.4$, $x = 0.4$



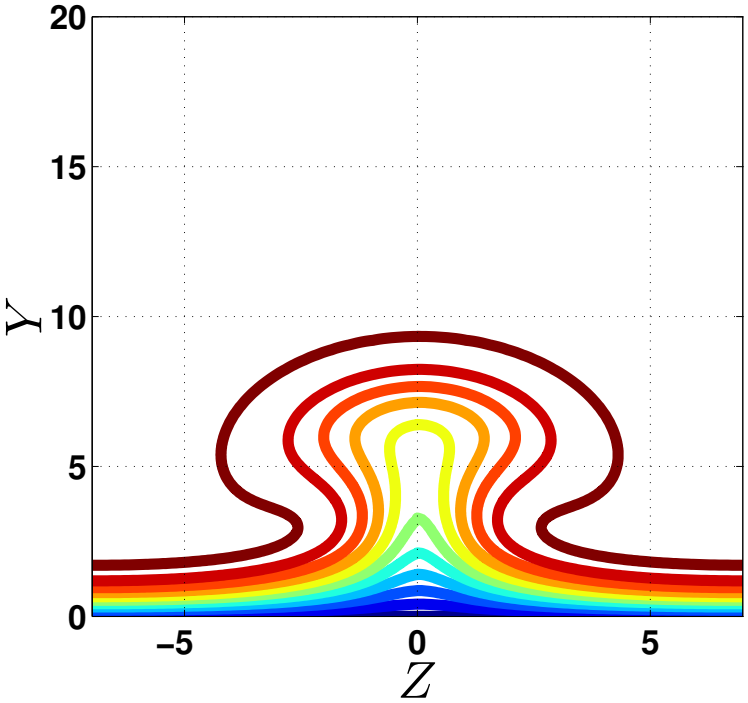
u constant streamwise contour lines



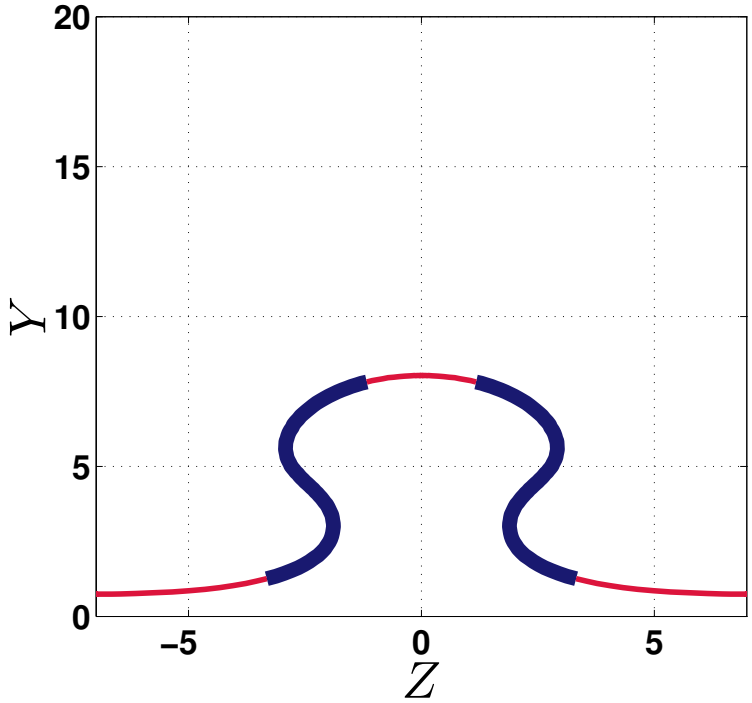
sections of particles trajectories starting $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Characteristics of the streaks $A_{s0} = 0.4$, $x = 1$



u constant streamwise contour lines

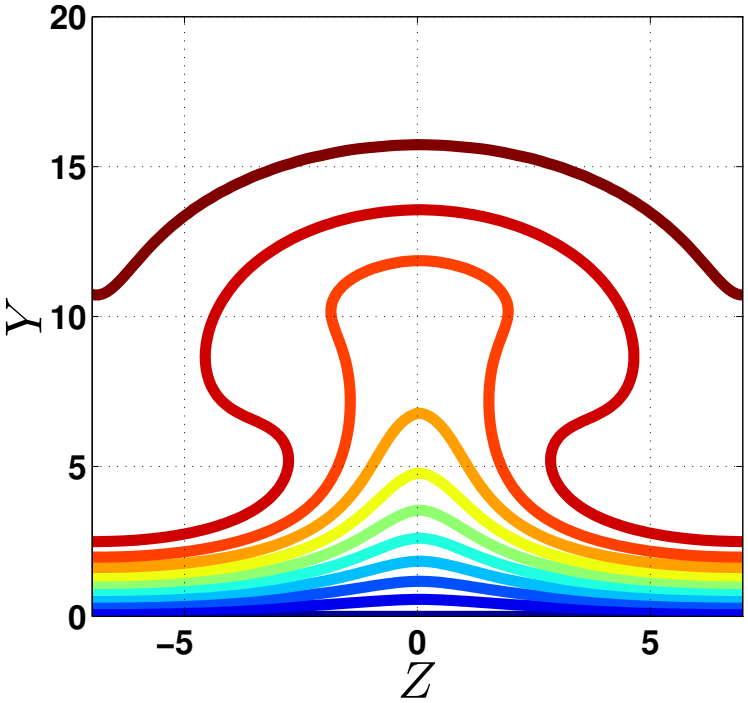


sections of particles trajectories

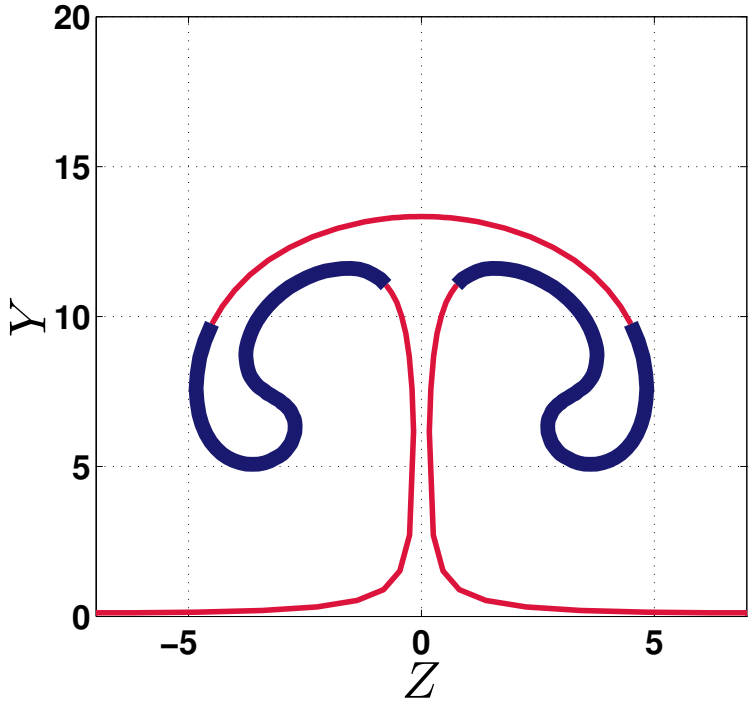
starting $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Characteristics of the streaks $A_{s0} = 0.4$, $x = 3$



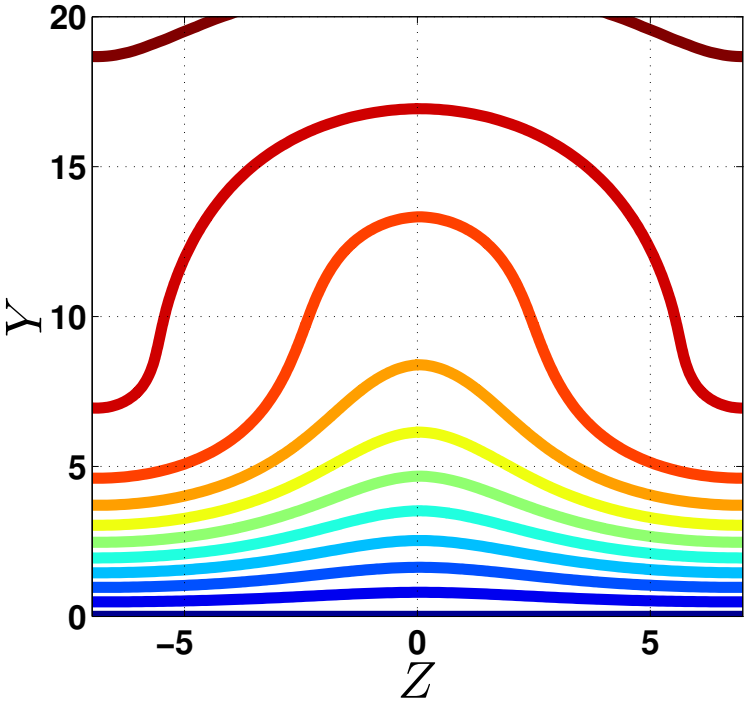
u constant streamwise contour lines



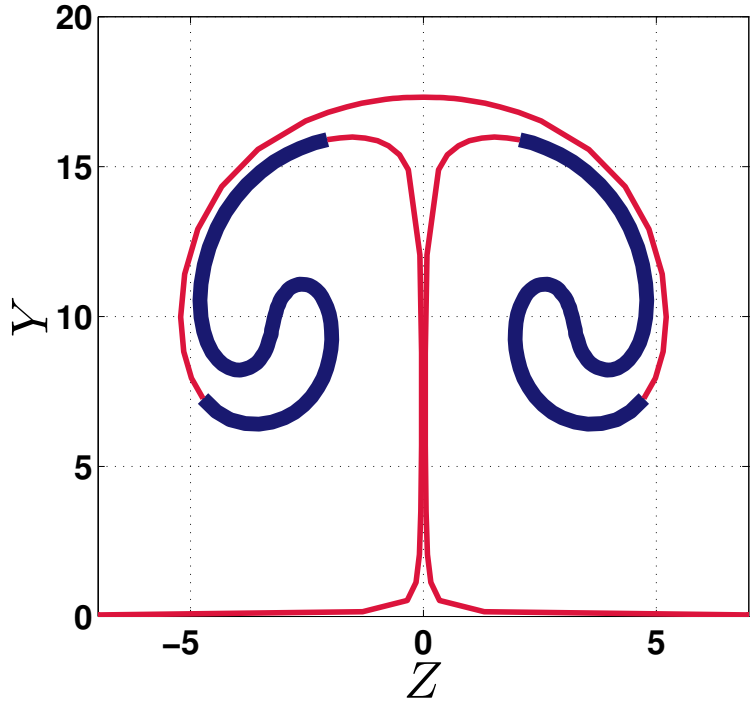
sections of particles trajectories starting $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Characteristics of the streaks $A_{s0} = 0.4$, $x = 7$



u constant streamwise contour lines

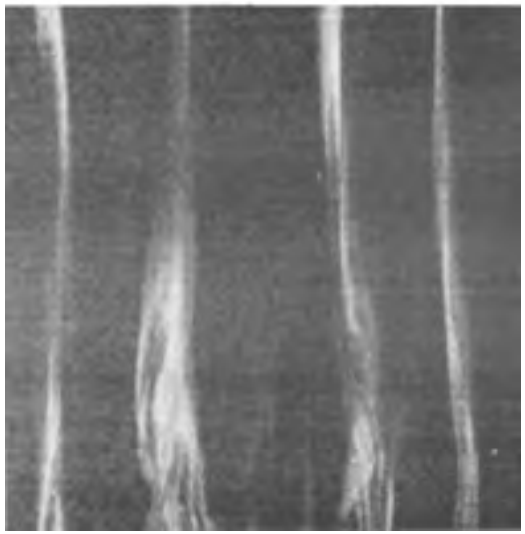


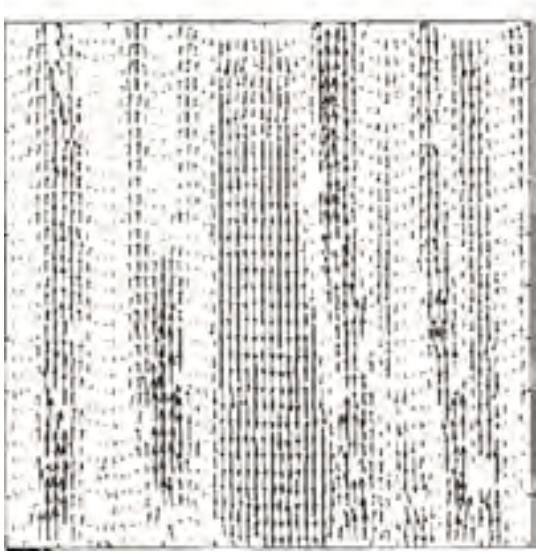
sections of particles trajectories

starting $x = 0.4$, $y = 3$

RNS 3D results: Transversal structure

Flow visualization

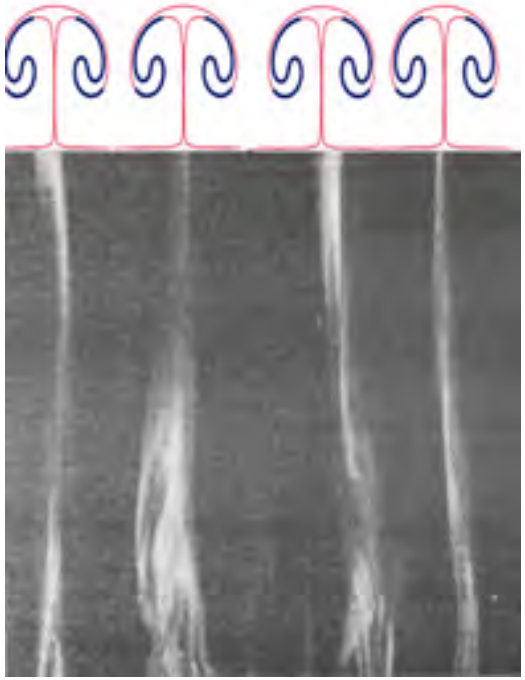


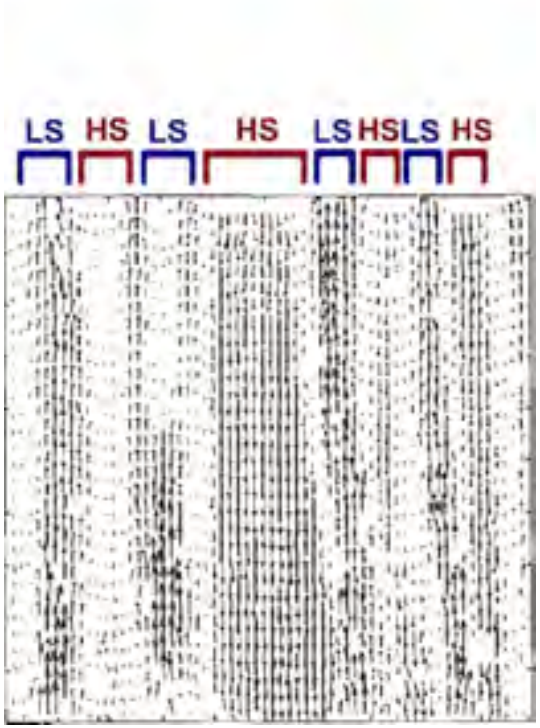


Alfredsson & Matsubara AIAA paper 2001

RNS 3D results: Transversal structure

Flow visualization

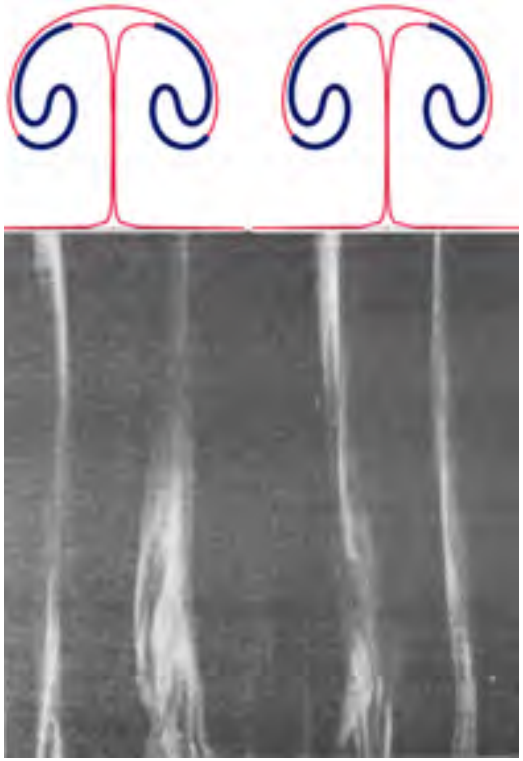


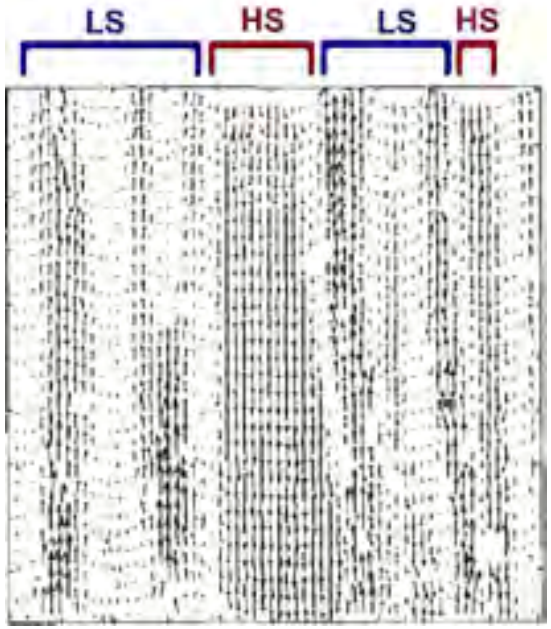


Alfredsson & Matsubara AIAA paper 2001

RNS 3D results: Transversal structure

Flow visualization





Alfredsson & Matsubara AIAA paper 2001

Two smoke traces per streak??

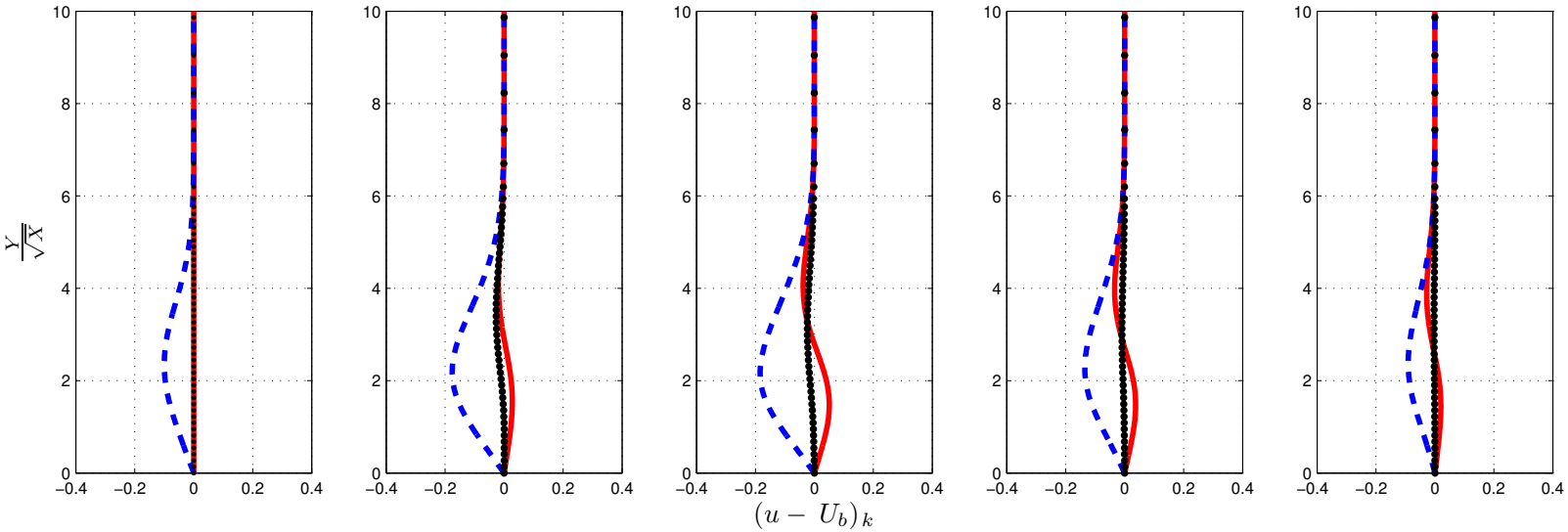
Discrepances in spanwise streak period

$\lambda_{expr} \sim \lambda_{LOT}/2$

Asai & Nishioka 1995, Asai & Konishi 2007

RNS 3D results: Effect on streamwise velocity

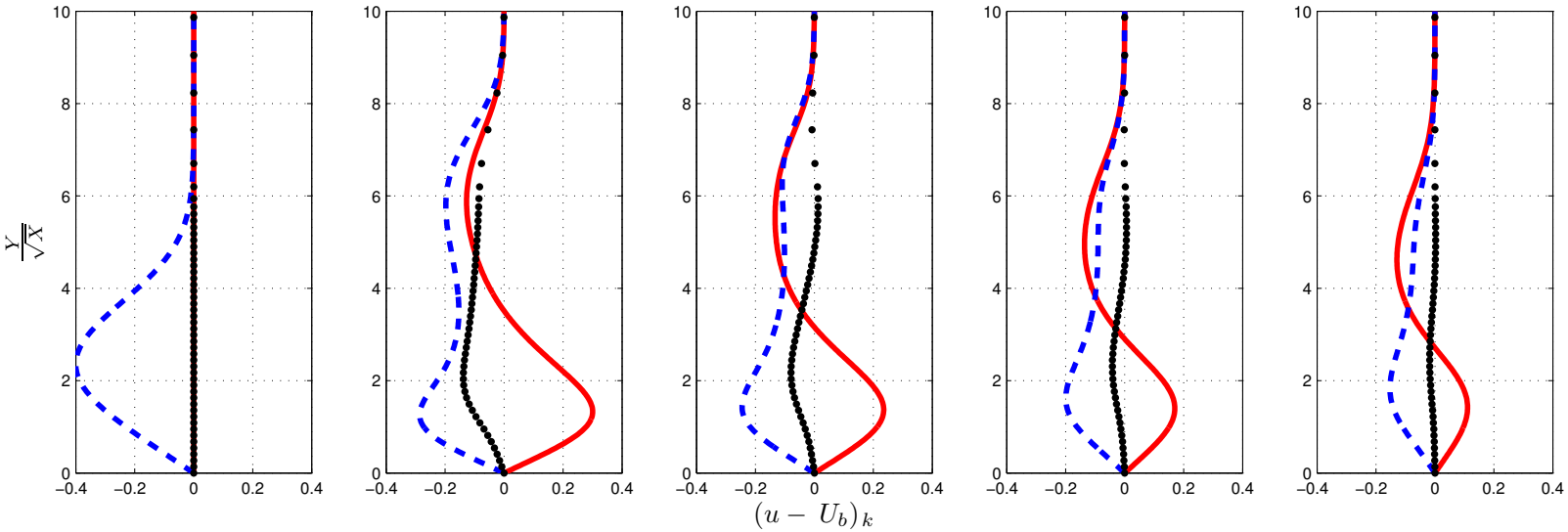
Spanwise Fourier modes of the streamwise velocity (substracting Blasius) $\mathbf{A_{s0} = 0.1}$



Mode $k = 0$, mode $k = 1$ and mode $k = 2$ at $\mathbf{x = 0.4, 1, 3, 5}$ and $\mathbf{7}$

RNS 3D results: Effect on streamwise velocity

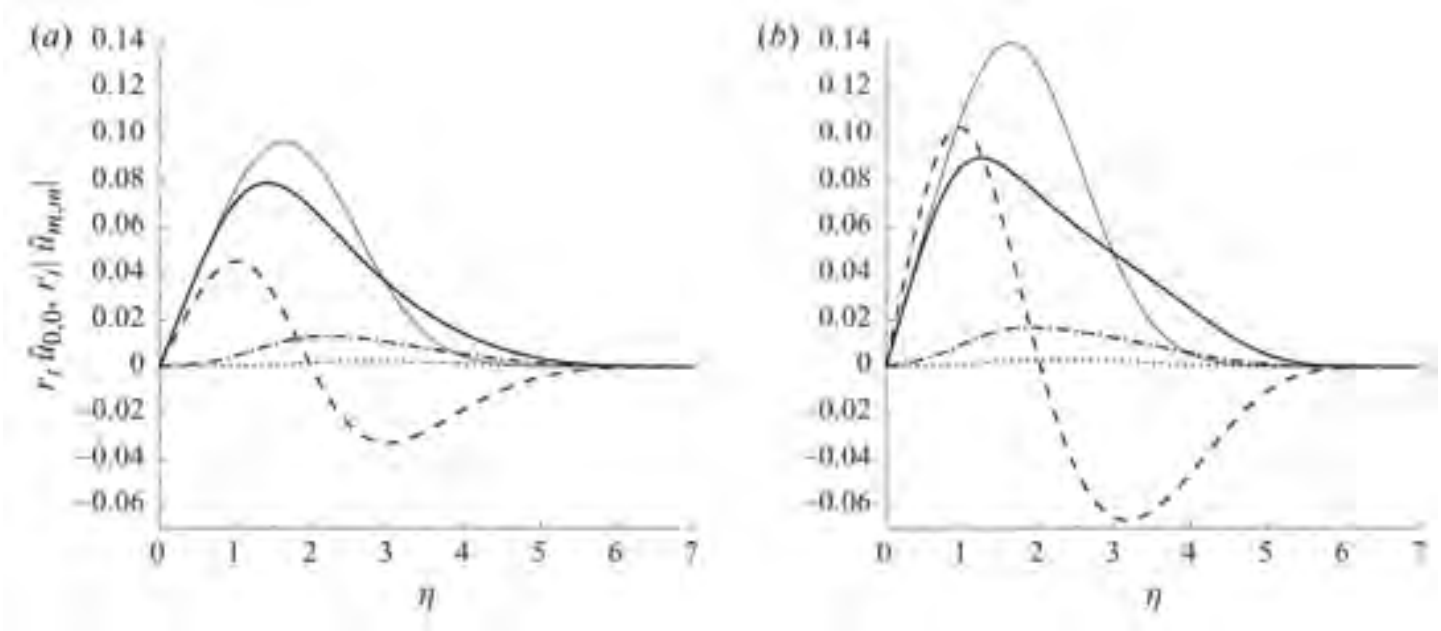
Spanwise Fourier modes of the streamwise velocity (substracting Blasius) $\mathbf{A_{s0} = 0.4}$



Mode $k = 0$, mode $k = 1$ and mode $k = 2$ at $\mathbf{x = 0.4, 1, 3, 5}$ and $\mathbf{7}$

RNS 3D results: Effect on streamwise velocity

Qualitative agreement with theorical analysis of non linear streaks



Ricco & Luo & Wu JFM 2011

RNS results: Computation of intrinsic NL streaks

- Eigenvalue problem, as $x \to 0$.

RNS results: Computation of intrinsic NL streaks

- Eigenvalue problem, as $x \longrightarrow 0$.
- Studied in **Luchini JFM (1996, 2000)** and **Higuera & Vega JFM (2009)**

RNS results: Computation of intrinsic NL streaks

- Eigenvalue problem, as $x \to 0$.
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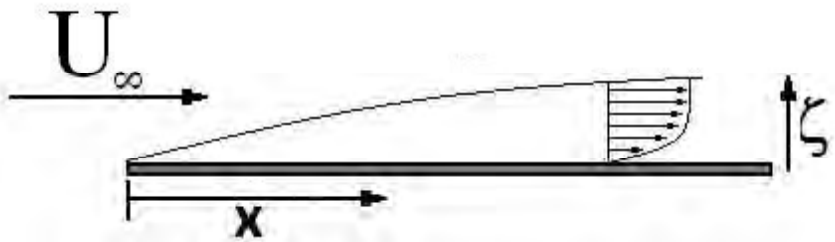
$$U_{le} = \beta^{1-\lambda} x^{1-\lambda} \tilde{U}(\zeta) \cos(z),$$
$$V_{le} = \beta^{1-\lambda} x^{1/2-\lambda} \tilde{V}(\zeta) \cos(z),$$
$$W_{le} = -\beta^{1/2-\lambda} x^{-\lambda} \left(\tilde{H}(\zeta) - \beta \sqrt{x} \tilde{V}(\zeta) \right) \sin(z),$$



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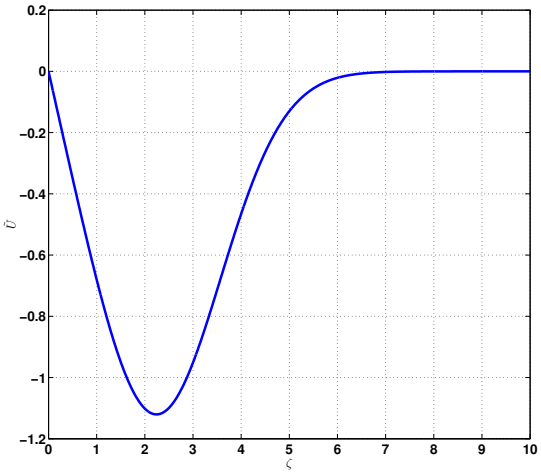
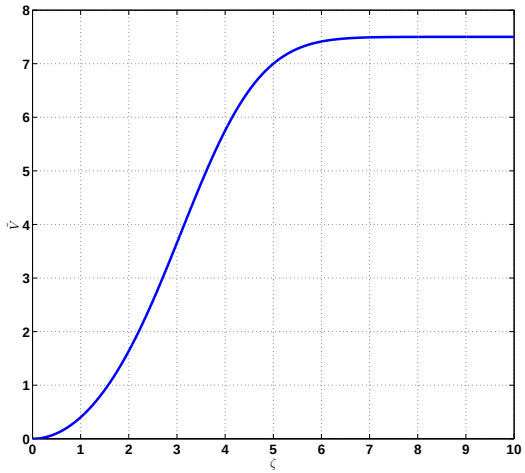
$$W_{le} = -\beta^{1/2-\lambda} x^{-\lambda} \left(\tilde{H}(\zeta) - \beta \sqrt{x} \tilde{V}(\zeta) \right) \sin(z),$$

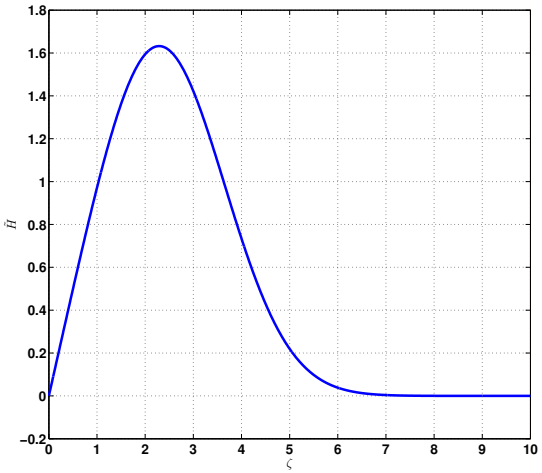
There is only one streamwise growing mode from the LE, for $\lambda = 0.7856$

RNS results: Computation of intrinsic NL streaks

- Eigenvalue problem, as $x \longrightarrow 0$.
- Studied in **Luchini JFM (1996, 2000)** and **Higuera & Vega JFM (2009)**







$\tilde{U}(\zeta)$, $\tilde{V}(\zeta)$ and $\tilde{H}(\zeta)$ profiles for the only growing mode

RNS results: *a* parameter family of NL streaks

Evolution of Blasius + Eigensolution

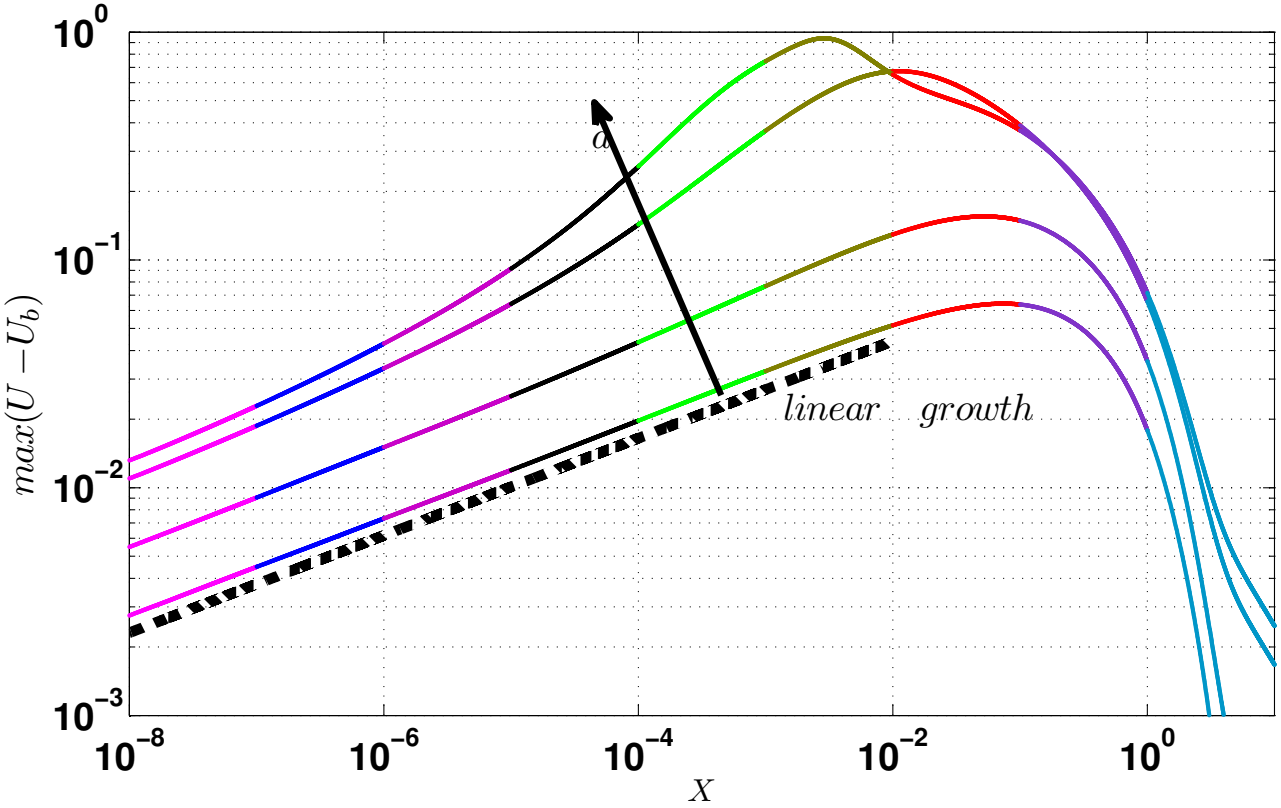
$$(u^0, v^0, w^0) = (U_b, V_b, 0) + a\,(U_{le}, V_{le}, W_{le}), \quad \beta = 1$$

RNS results: a parameter family of NL streaks

Evolution of Blasius + Eigensolution

$$(u^0, v^0, w^0) = (U_b, V_b, 0) + a\,(U_{le}, V_{le}, W_{le}), \quad \beta = 1$$

Streamwise velocity, maximum deviation from Blasius profile



a = 0.125

a = 0.250

a = 0.500

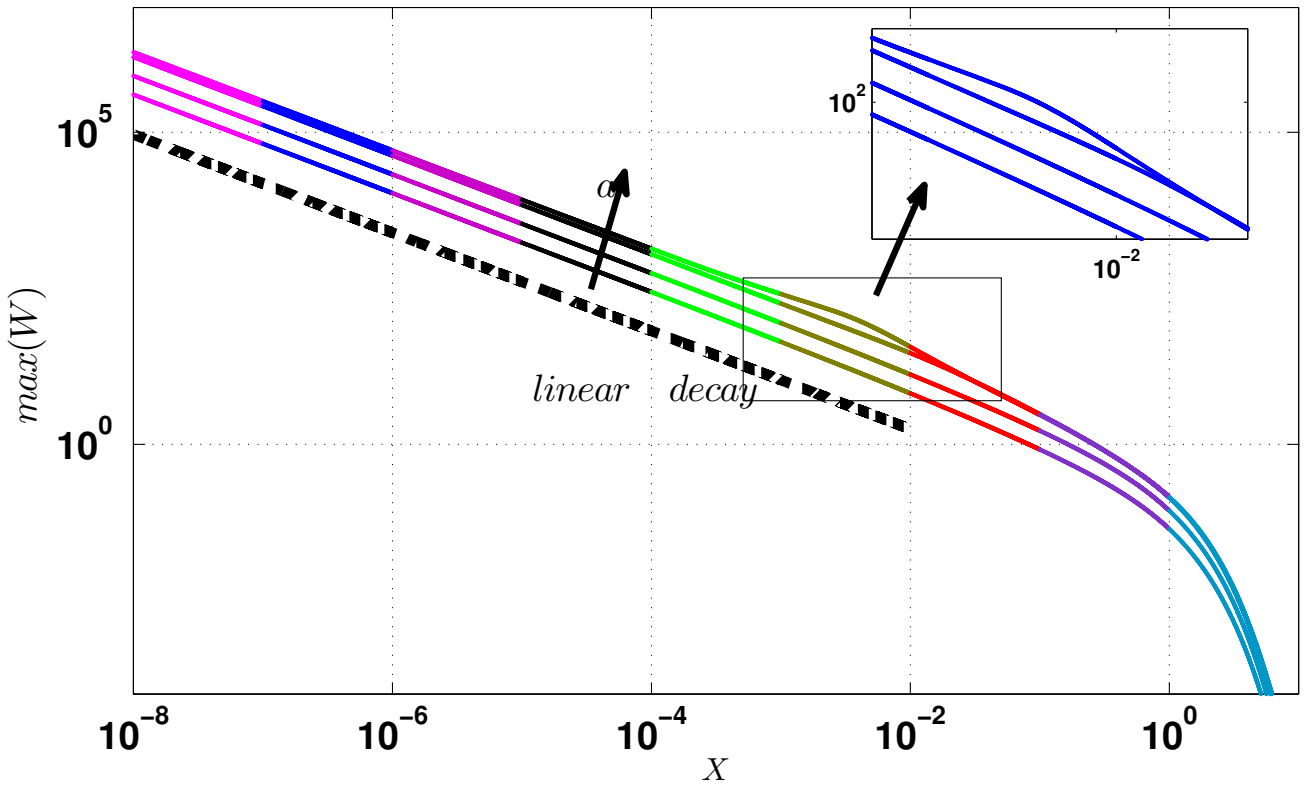
a = 0.600

RNS results: a parameter family of NL streaks

Evolution of Blasius + Eigensolution

$$(u^0, v^0, w^0) = (U_b, V_b, 0) + a\,(U_{le}, V_{le}, W_{le}), \quad \beta = 1$$

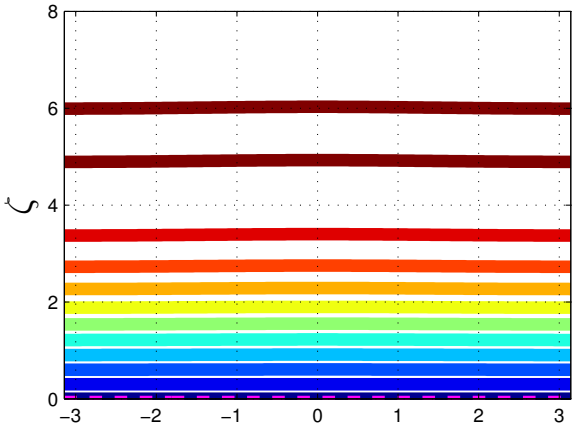
Spanwise velocity, maximum value



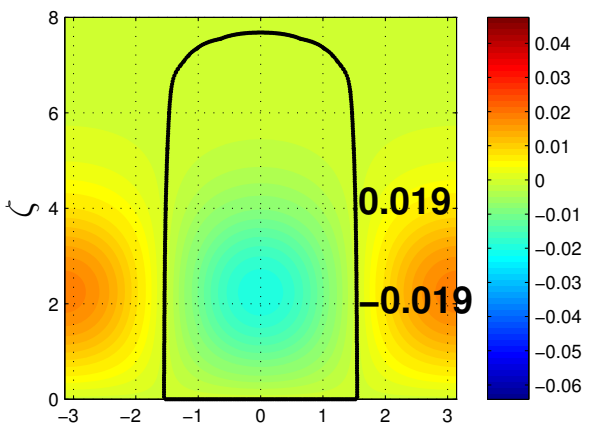
a = 0.125
a = 0.250
a = 0.500
a = 0.600

RNS 3D results: Transversal structure

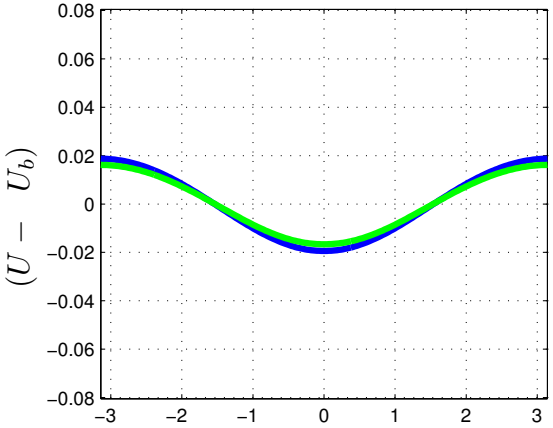
Characteristics of the streamwise velocity u
$\mathbf{a = 0.125}$, $\mathbf{x = 1 \cdot 10^{-4}}$



constant contour lines



deviation from Blasius



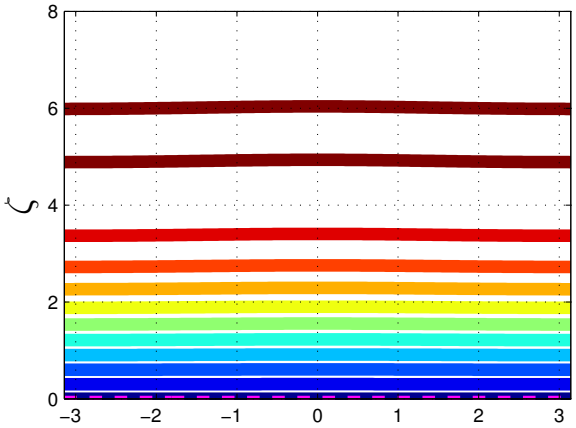
spanwise variation

$\zeta = 2$

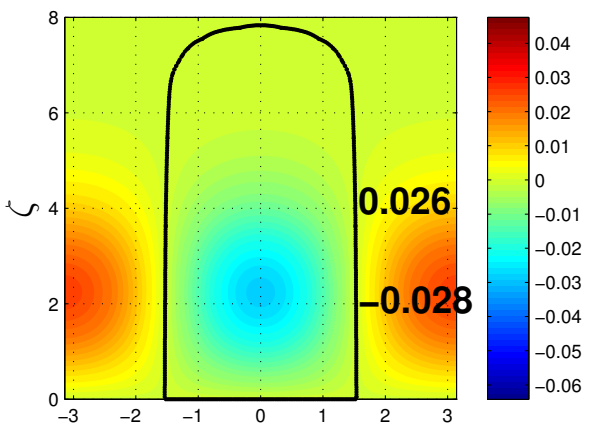
$\zeta = 3$

RNS 3D results: Transversal structure

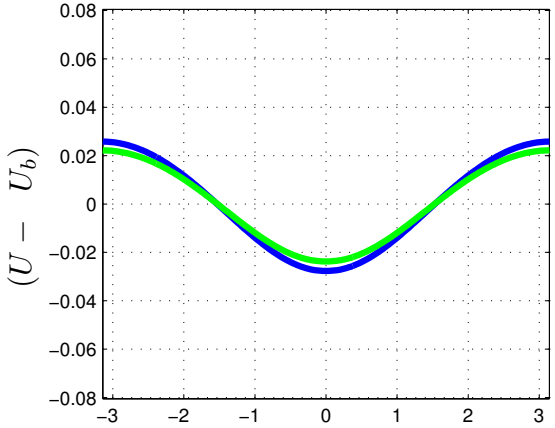
Characteristics of the streamwise velocity u
$\mathbf{a = 0.125}$, $\mathbf{x = 5 \cdot 10^{-4}}$



constant contour lines



deviation from Blasius



spanwise variation

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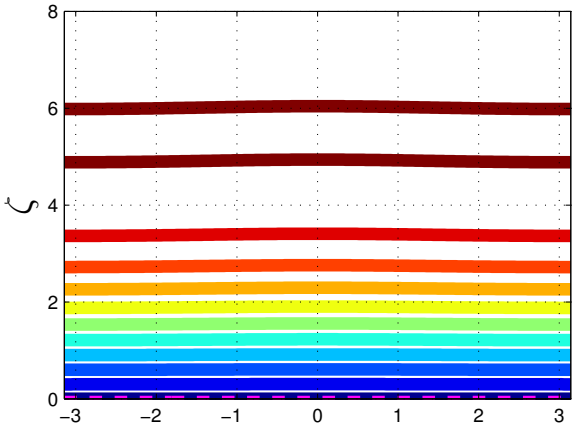
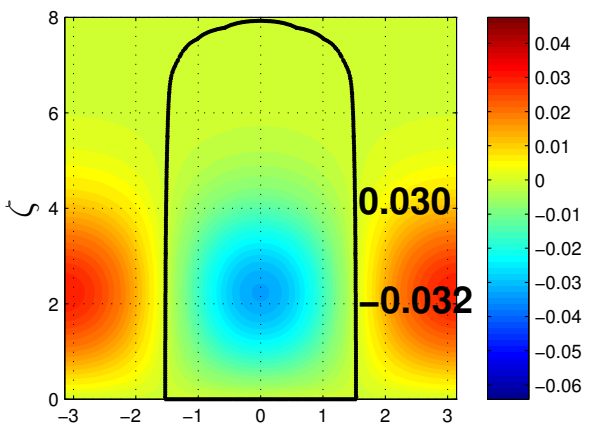
$\zeta = 3$

RNS 3D results: Transversal structure

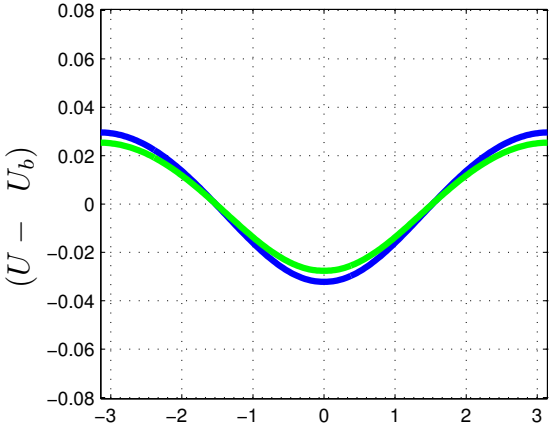
Characteristics of the streamwise velocity u
$\mathbf{a = 0.125}$, $\mathbf{x = 1 \cdot 10^{-3}}$



constant contour lines



deviation from Blasius



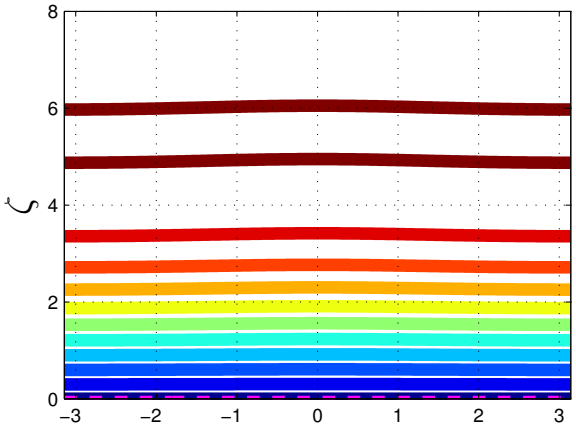
spanwise variation

$\zeta = 2$

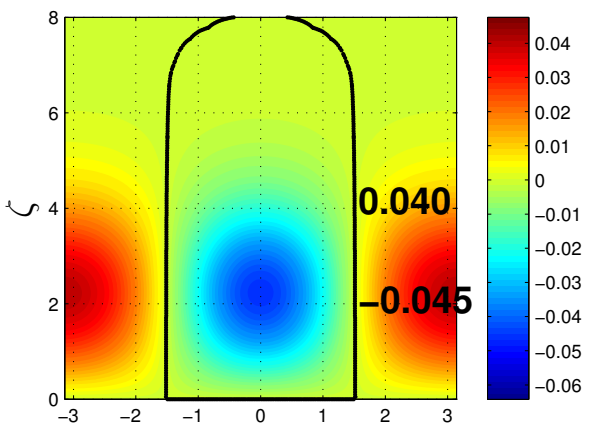
$\zeta = 3$

RNS 3D results: Transversal structure

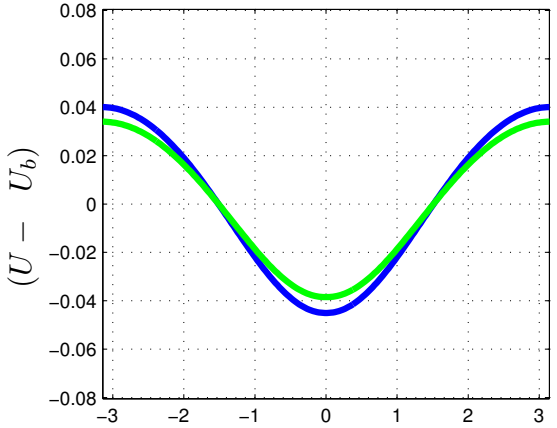
Characteristics of the streamwise velocity u
$\mathbf{a = 0.125}$, $\mathbf{x = 5 \cdot 10^{-3}}$



constant contour lines



deviation from Blasius



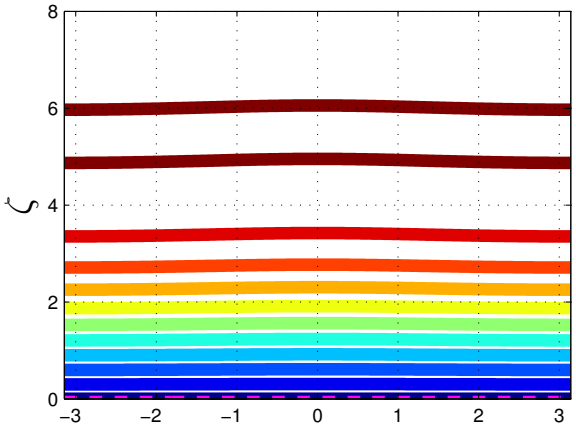
spanwise variation

$\zeta = 2$

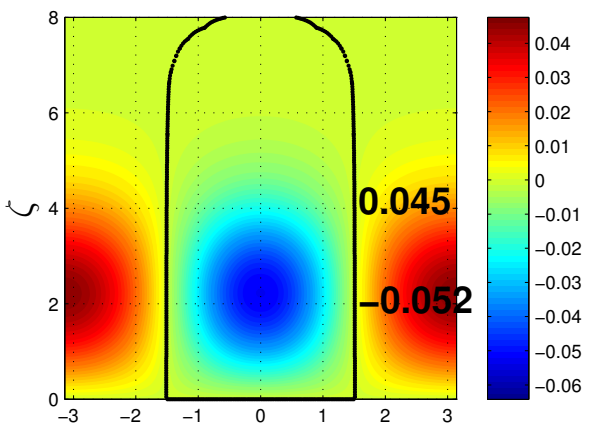
$\zeta = 3$

RNS 3D results: Transversal structure

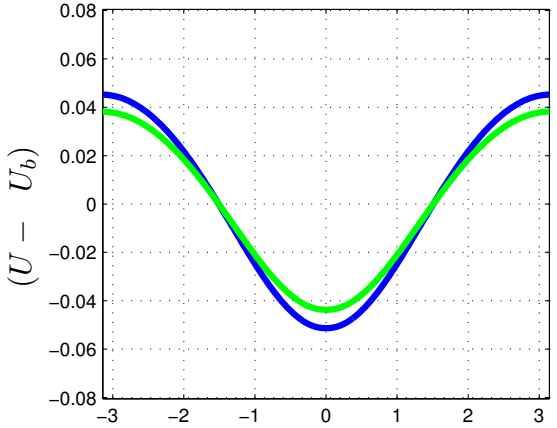
Characteristics of the streamwise velocity u
$\mathbf{a = 0.125}$, $\mathbf{x = 1 \cdot 10^{-2}}$



constant contour lines



deviation from Blasius



spanwise variation

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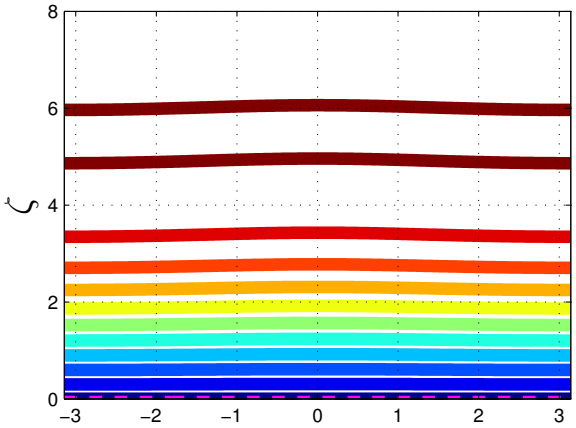
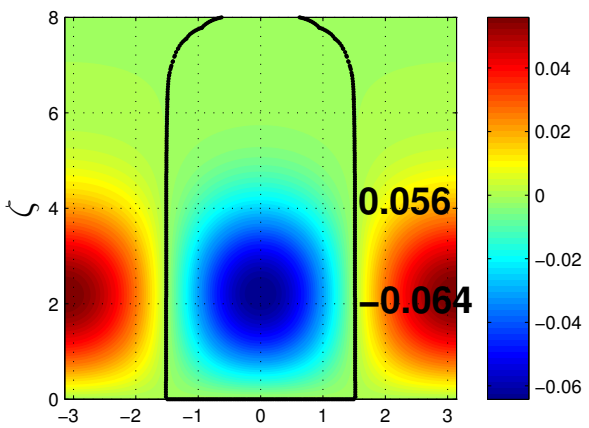
$\zeta = 3$

RNS 3D results: Transversal structure

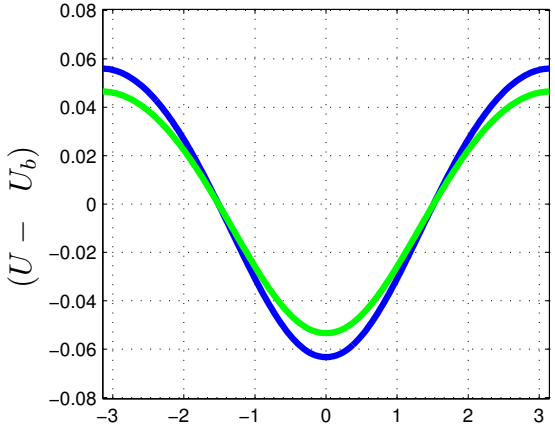
Characteristics of the streamwise velocity u
$\mathbf{a = 0.125}$, $\mathbf{x = 5 \cdot 10^{-2}}$



constant contour lines



deviation from Blasius



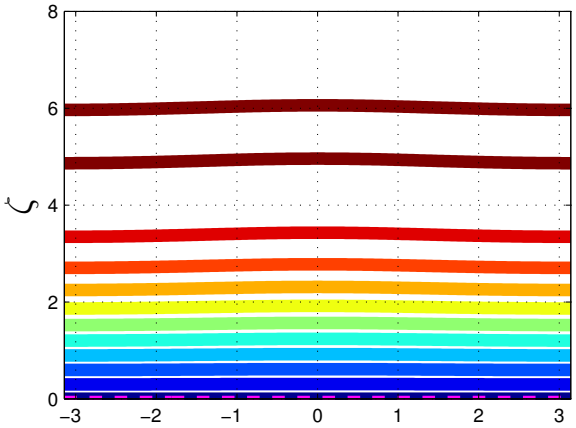
spanwise variation

$\zeta = 2$

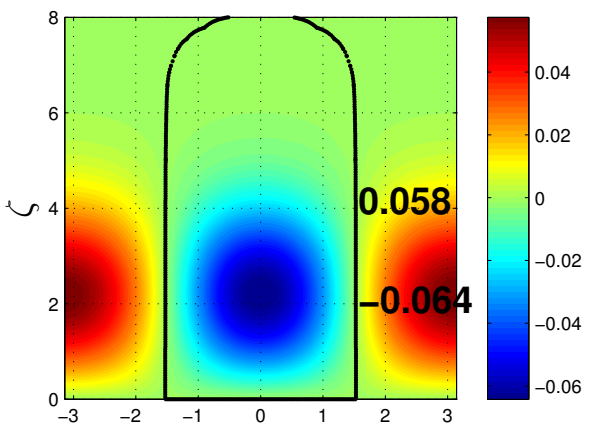
$\zeta = 3$

RNS 3D results: Transversal structure

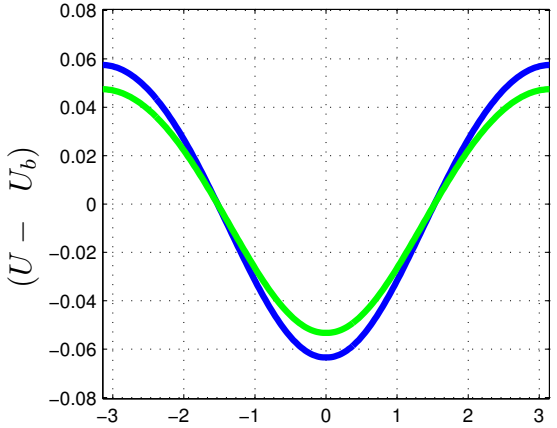
Characteristics of the streamwise velocity u
$\mathbf{a = 0.125}$, $\mathbf{x = 1 \cdot 10^{-1}}$



constant contour lines



deviation from Blasius



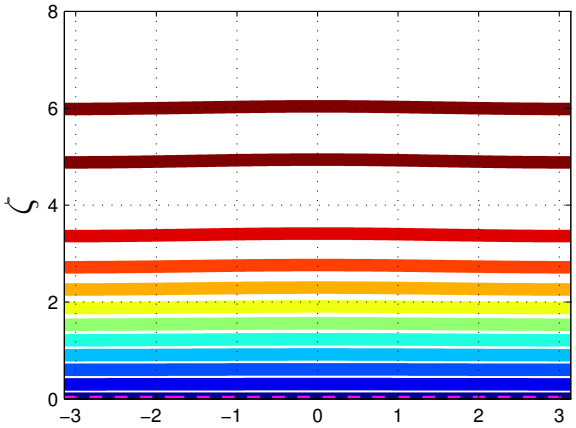
spanwise variation

$\zeta = 2$

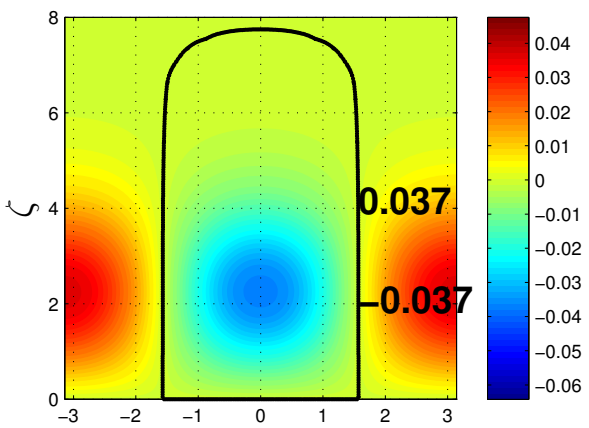
$\zeta = 3$

RNS 3D results: Transversal structure

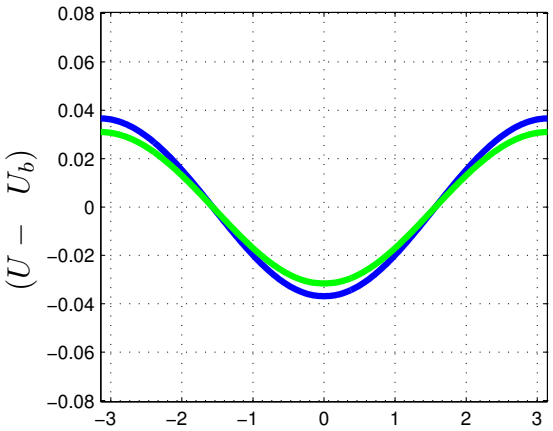
Characteristics of the streamwise velocity u
$\mathbf{a = 0.125}$, $\mathbf{x = 5 \cdot 10^{-1}}$



constant contour lines



deviation from Blasius



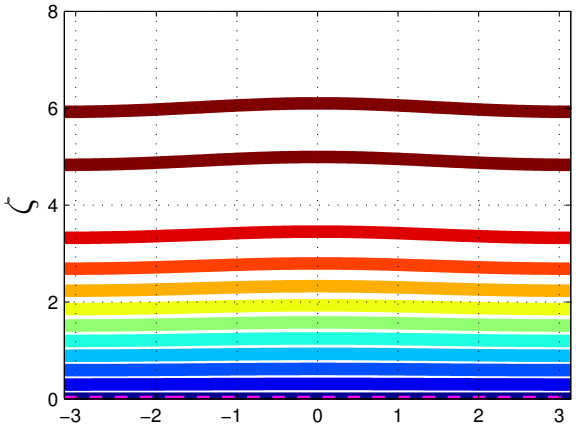
spanwise variation

$\zeta = 2$

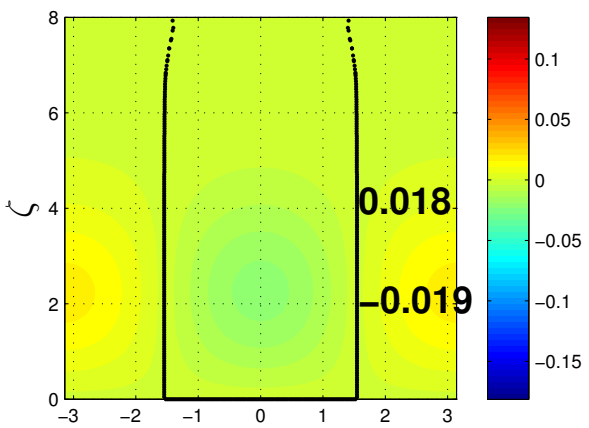
$\zeta = 3$

RNS 3D results: Transversal structure

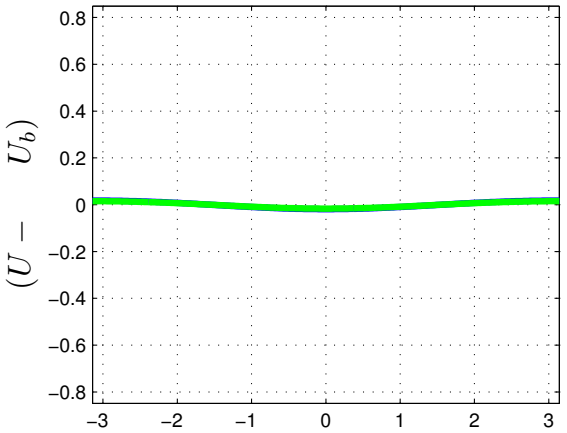
Characteristics of the streamwise velocity u
$\mathbf{a = 0.6}$, $\mathbf{x = 5 \cdot 10^{-8}}$



constant contour lines



deviation from Blasius



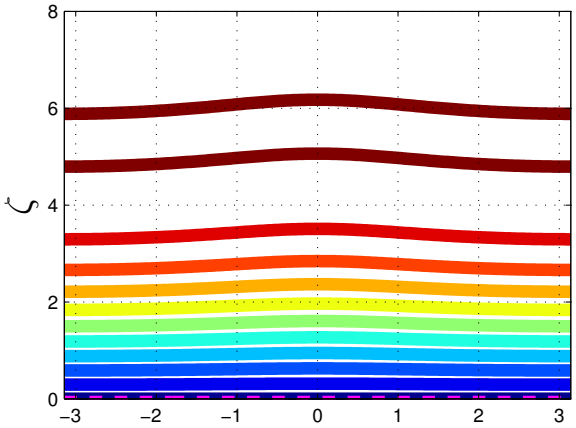
spanwise variation

$\zeta = 2$

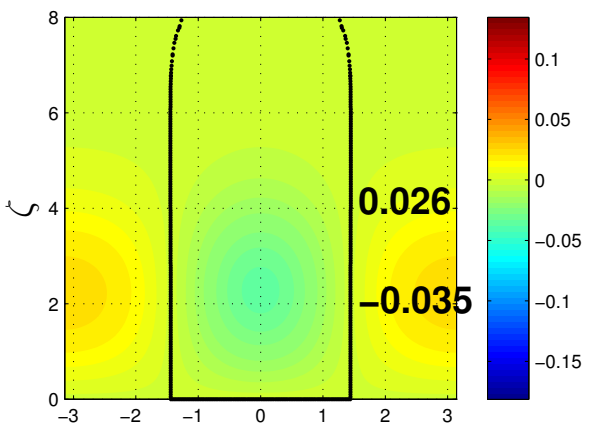
$\zeta = 3$

RNS 3D results: Transversal structure

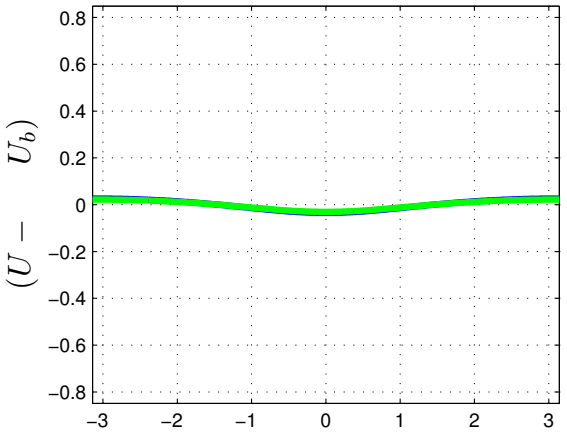
Characteristics of the streamwise velocity u
$\mathbf{a = 0.6,\ x = 5 \cdot 10^{-7}}$



constant contour lines



deviation from Blasius



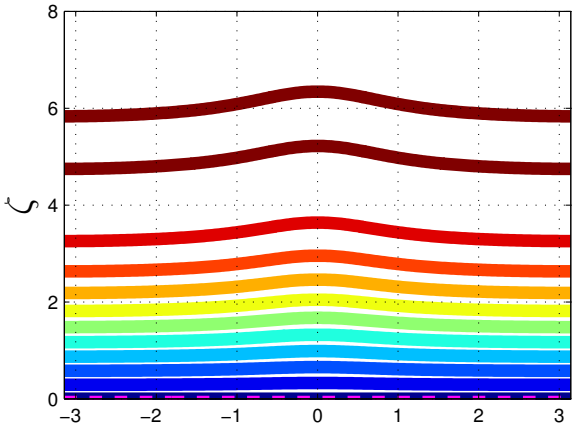
spanwise variation

$\zeta = 2$

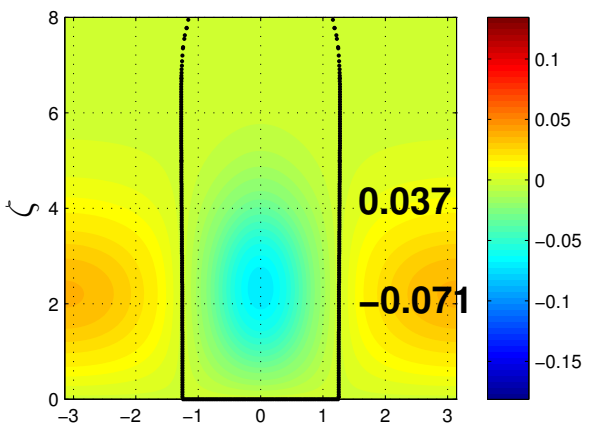
$\zeta = 3$

RNS 3D results: Transversal structure

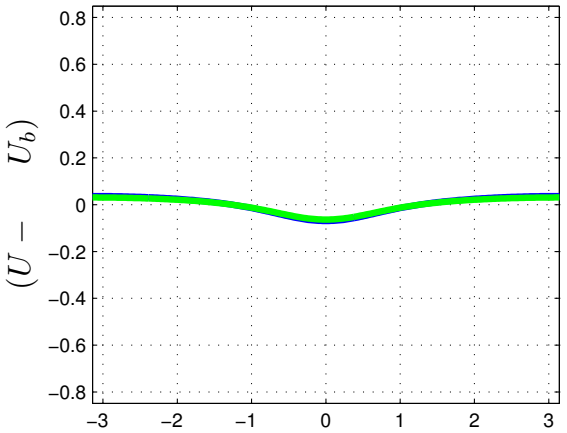
Characteristics of the streamwise velocity u
$\mathbf{a = 0.6}$, $\mathbf{x = 5 \cdot 10^{-6}}$



constant contour lines



deviation from Blasius



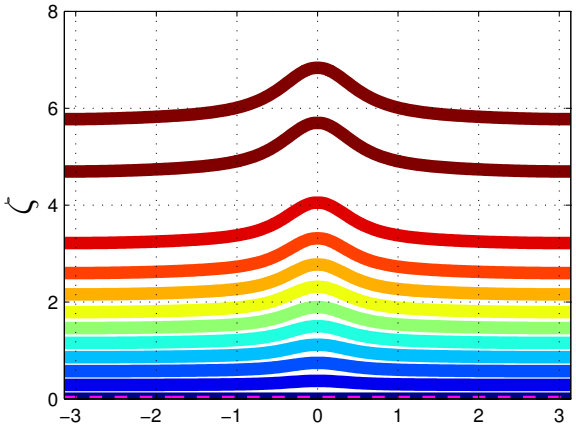
spanwise variation

$\zeta = 2$

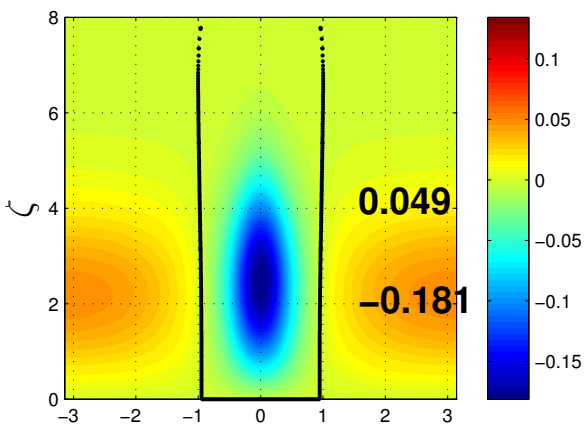
$\zeta = 3$

RNS 3D results: Transversal structure

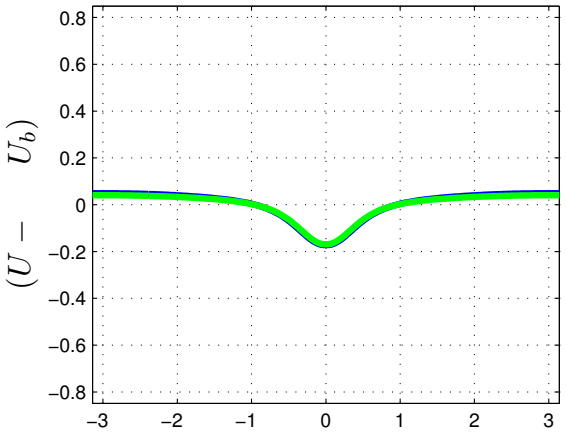
Characteristics of the streamwise velocity u
$\mathbf{a = 0.6,\ x = 5 \cdot 10^{-5}}$



constant contour lines



deviation from Blasius



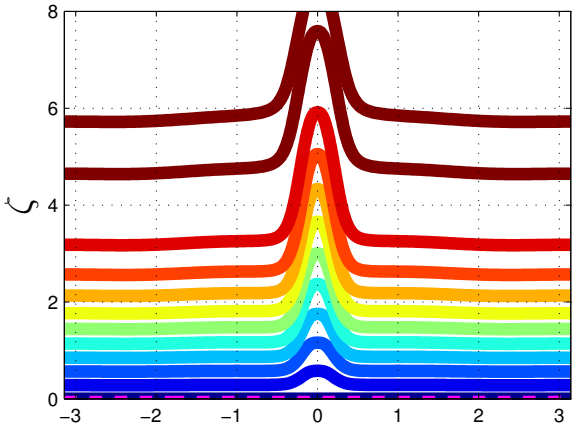
spanwise variation

$\zeta = 2$

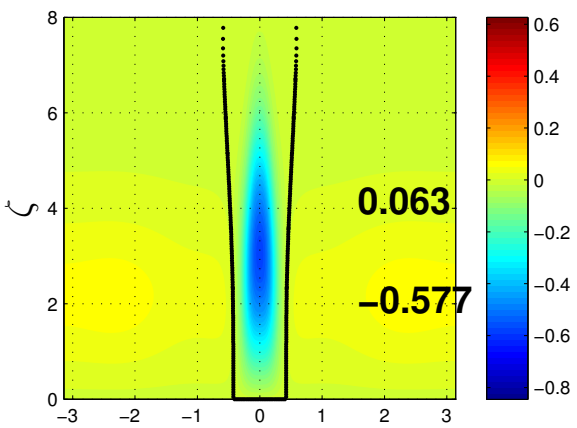
$\zeta = 3$

RNS 3D results: Transversal structure

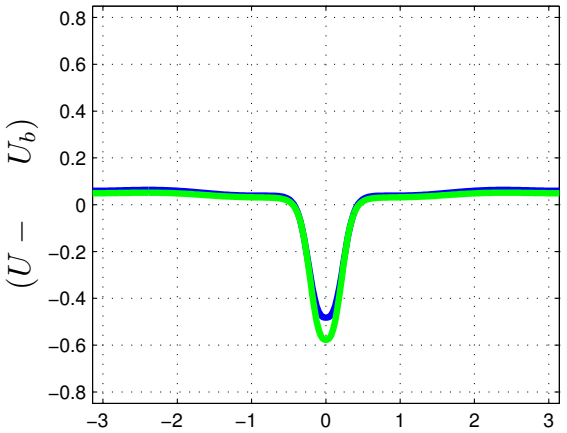
Characteristics of the streamwise velocity u
$\mathbf{a = 0.6}$, $\mathbf{x = 5 \cdot 10^{-4}}$



constant contour lines



deviation from Blasius



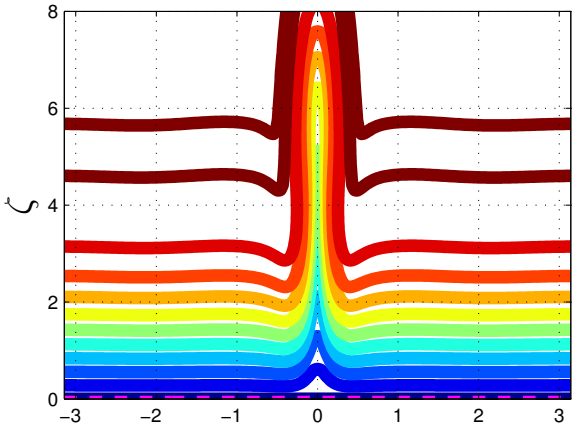
spanwise variation

$\zeta = 2$

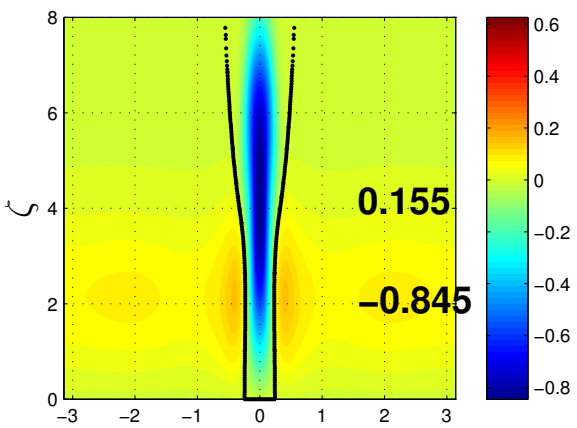
$\zeta = 3$

RNS 3D results: Transversal structure

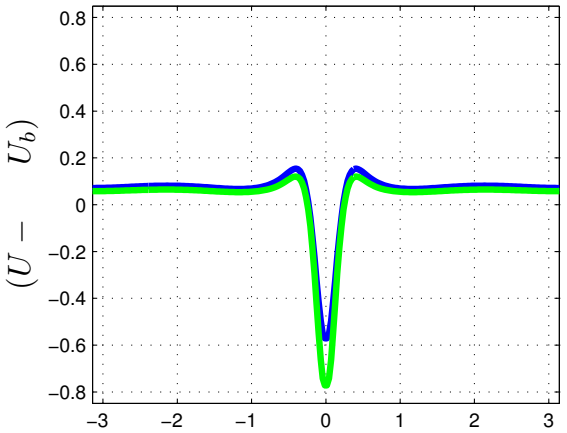
Characteristics of the streamwise velocity u
$\mathbf{a = 0.6,\ x = 5 \cdot 10^{-3}}$



constant contour lines



deviation from Blasius



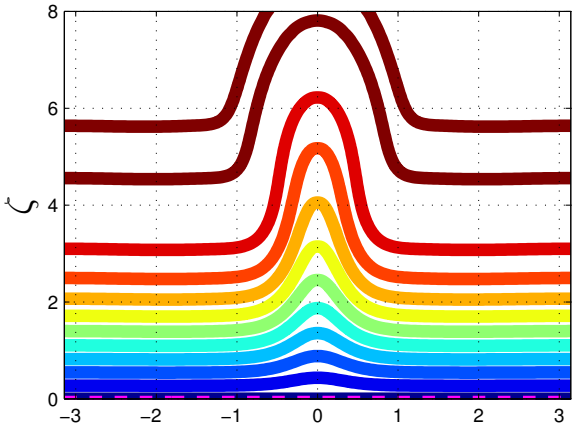
spanwise variation

$\zeta = 2$

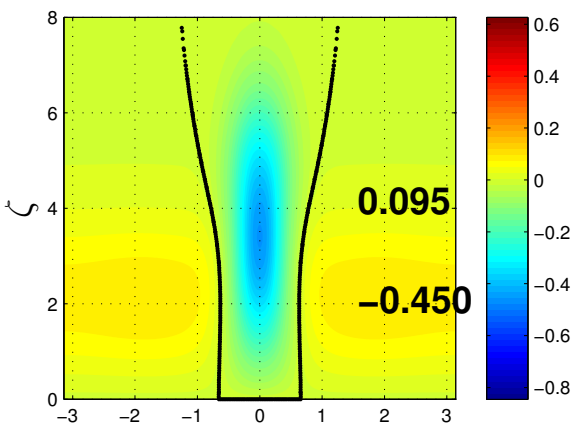
$\zeta = 3$

RNS 3D results: Transversal structure

Characteristics of the streamwise velocity u
$\mathbf{a = 0.6,\ x = 5 \cdot 10^{-2}}$



constant contour lines



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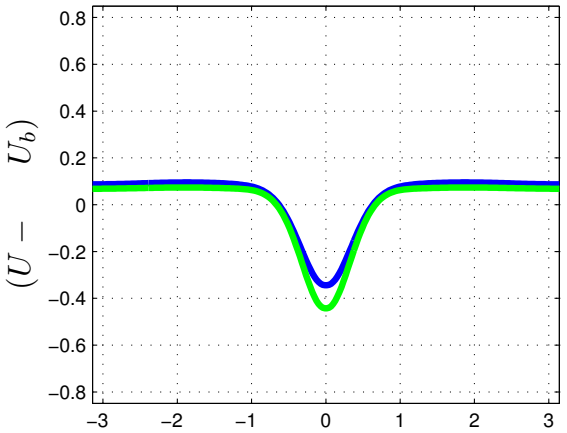


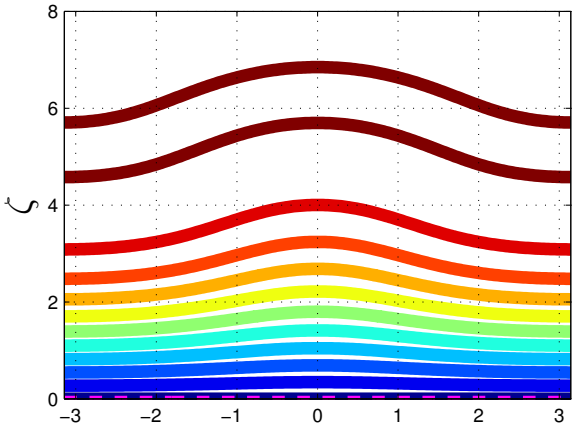
spanwise variation

$\zeta = 2$

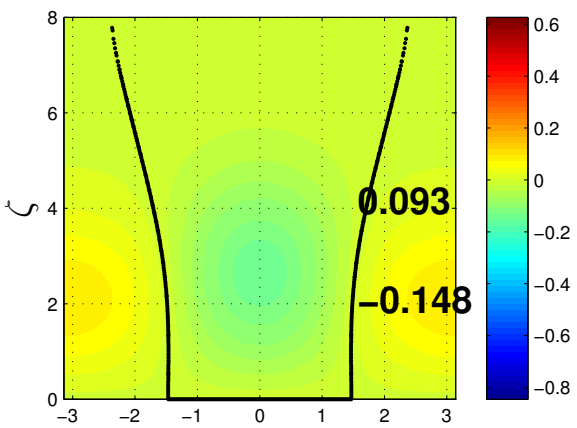
$\zeta = 3$

RNS 3D results: Transversal structure

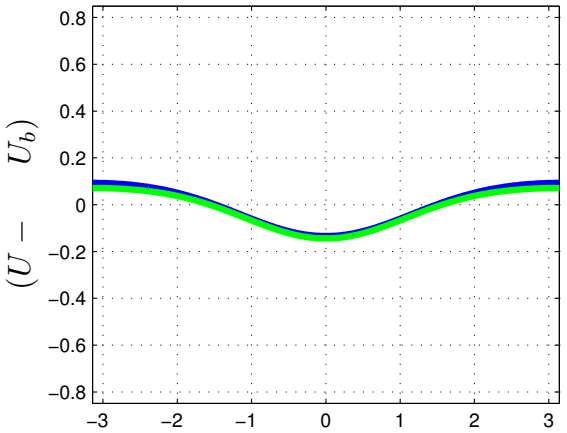
Characteristics of the streamwise velocity u
$\mathbf{a = 0.6,\ x = 5 \cdot 10^{-1}}$



constant contour lines



deviation from Blasius



spanwise variation

$\zeta = 2$

$\zeta = 3$

Final remarks

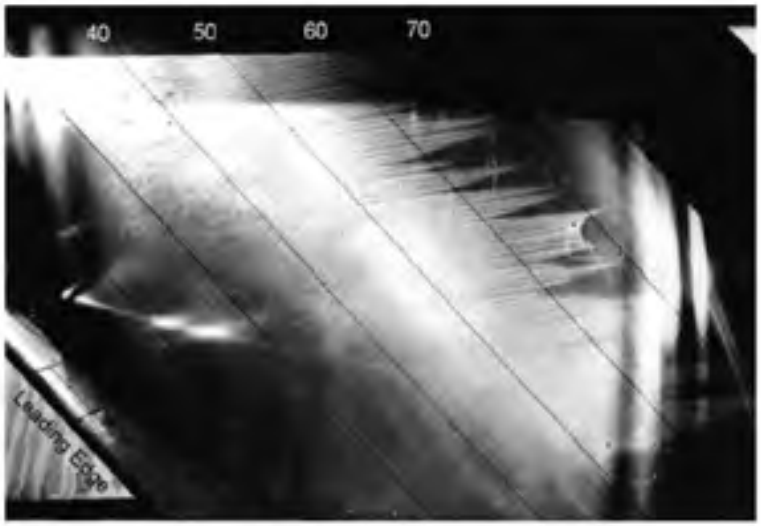
- RNS equations describe the downstream evolution of nonlinear streaks
  - Much less computational cost than DNS
  - More robust than PSE
- Motion in transversal plane essential to understand streak flow configuration
  - Transversal counter-rotating motion (Two smokes traces per streak??)
  - Strongly non linear effect in streamwise velocity profile
  - Not detected in linear or small amplitude streak analysis
- RNS computations of intrinsic nonlinear streaks
  - Starting with the LE asymptotic profile (growing mode)
  - For small a, linear theory results recovered
  - For $a \sim 1$ show nonlinear distorted profile

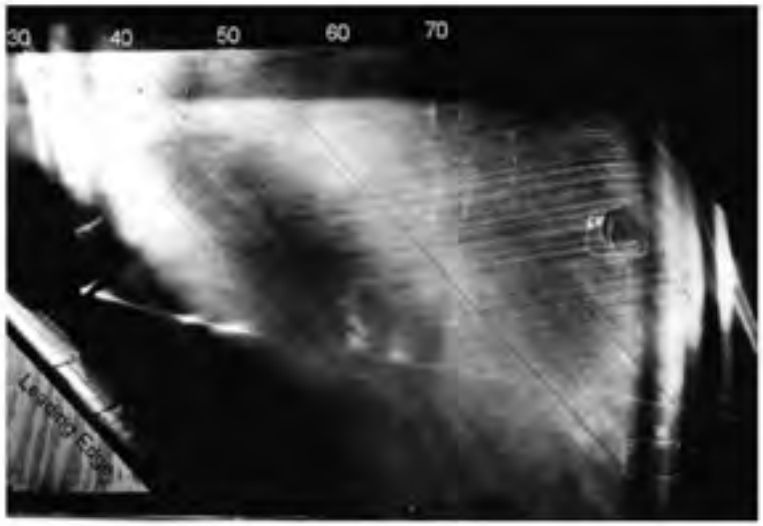
Final remarks

- RNS would be probably useful to other flow configurations with two short and one long scale
  - Crossflow vortices in a swept wing
  - Edge states in plane Couette flow

X-flow Vortices

- Use RNS code to explore stability and interaction with grooves of X-flow vortices:

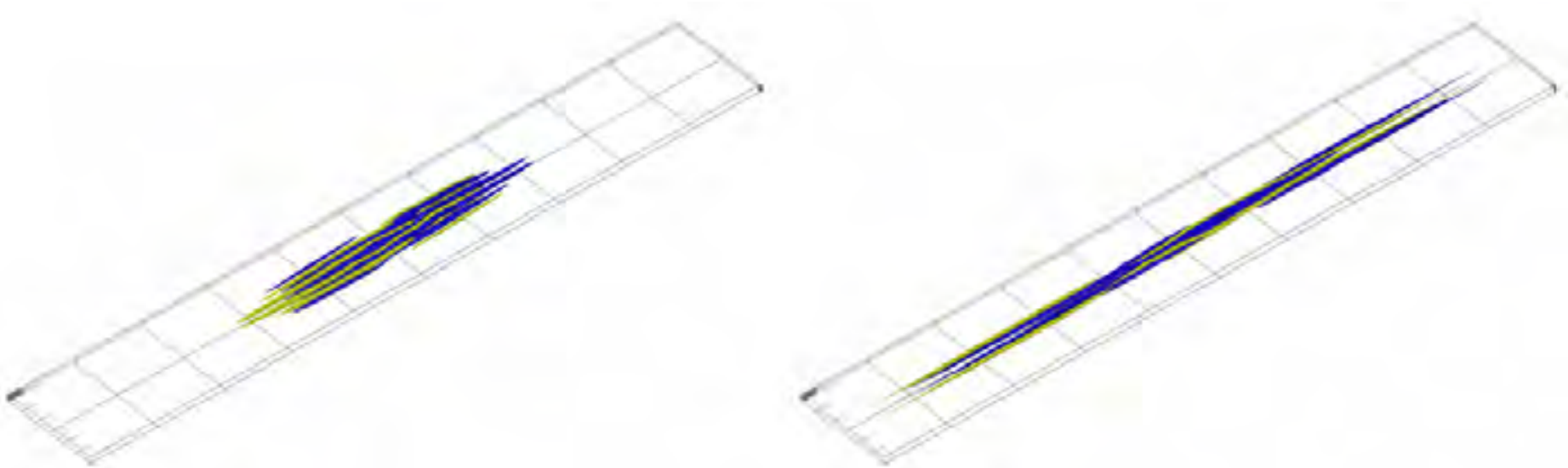




Saric & Reed AIAA 2003

Edge states

- RNS for edge states in Couette/Poiseuille flow at $Re \to \infty$. Couette flow (x-localized) $Re = 375$ and $Re = 1000$.



Duguet & Schlatter & Henningson PoF 2009

Thanks for your attention!